Strong and weak localizations: experimental study with the atomic kicked rotor

Jean-François Clément
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Quantum Chaos team

Outline

Anderson localization

Quasiperiodic kicked rotor

Recent results on weak localization

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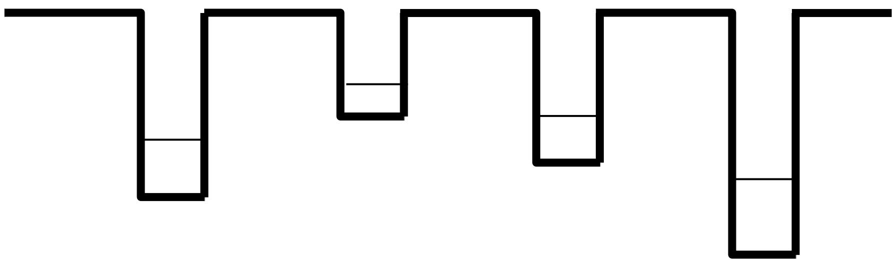
Anderson localization

Quasiperiodic kicked rotor

Recent results on weak localization

Anderson model (1958)

Tight-binding model + disorder



$$H = \sum_n V_n |n\rangle\langle n| + \sum_{n\neq m} t_{nm} |n\rangle\langle m|$$

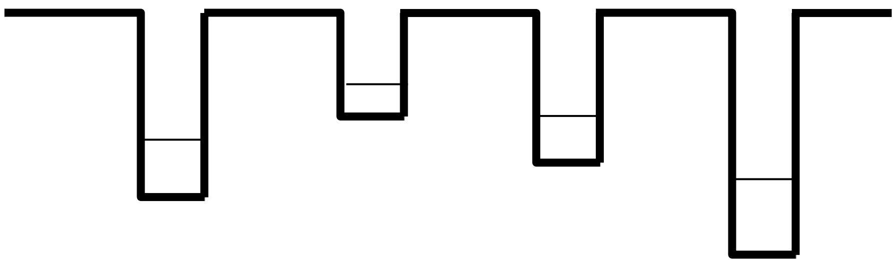
Disorder
Randomly distributed over [-W/2, W/2]

Hopping
between site n and site m

P. W. Anderson, Absence of diffusion in a certain random lattices, PRL **109** 1492-1505 (1958)

Anderson model (1958)

Tight-binding model + disorder



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P. W. Anderson, Absence of diffusion in a certain random lattices, PRL **109** 1492-1505 (1958)

→ 1D, 2D systems : eigenstates localized in position space

→ 3D system : phase transition

- $\frac{W}{t} \gg 1$ ***Localization***
- $\frac{W}{t} \ll 1$ ***Diffusion***

E. Abrahams et al, Scaling Theory of Localization: Absence of Quantum Diffusion in Two Dimensions PRL **42** 673-676 (1979)

Scaling theory (1979)

Study the change of the generalized dimensionless conductance g with the typical size given by L

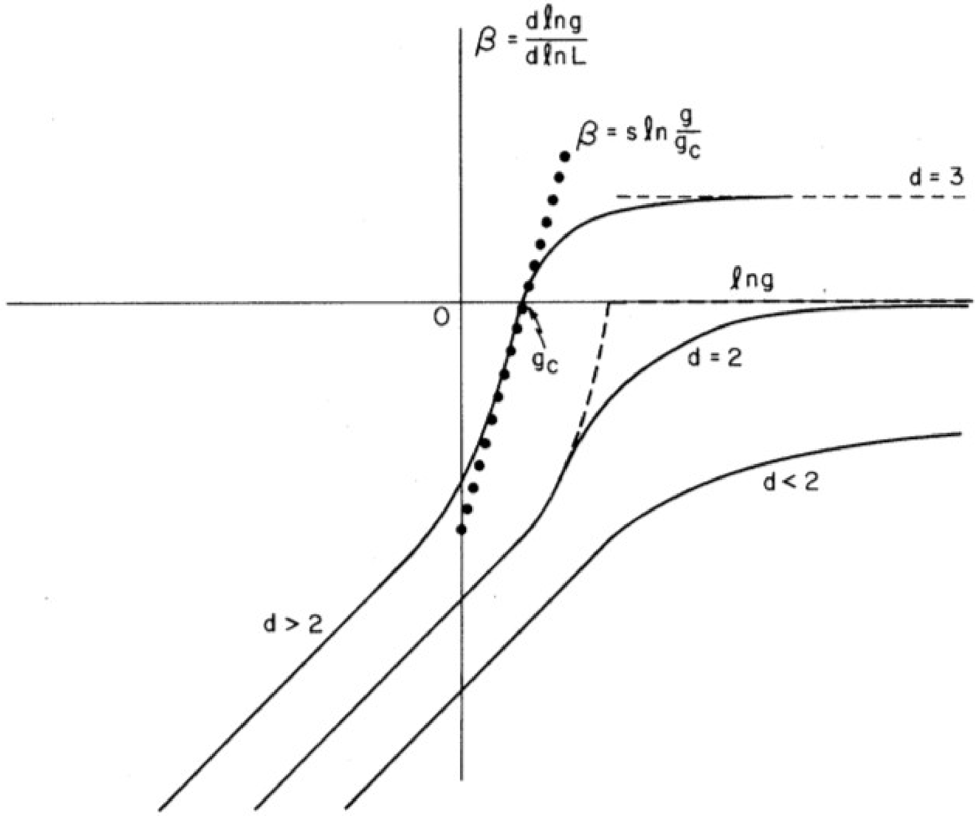
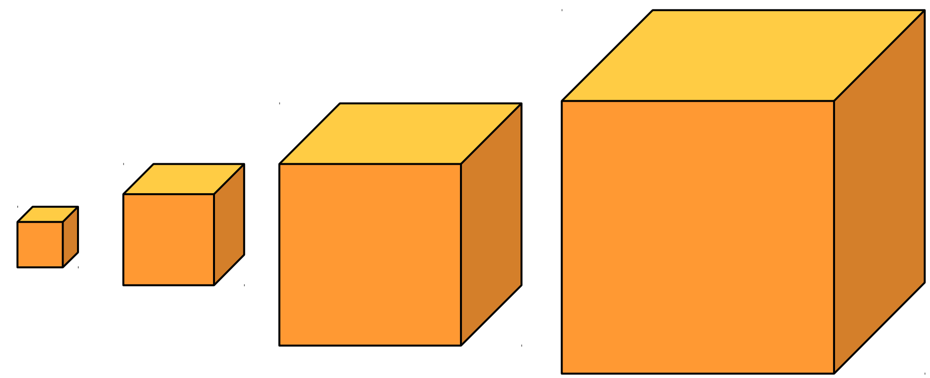


FIG. 1. Plot of $\beta(g)$ vs $\ln g$ for $d>2$, $d=2$, $d<2$. $g(L)$ is the normalized "local conductance." The approximation $\beta = s\ln(g/g_c)$ is shown for $g>2$ as the solid-circled line; this unphysical behavior necessary for a conductance jump in $d=2$ is shown dashed.



d = 1 , β < 0: localization
d = 2 , β < 0: localization
d = 3 : metal-insulator transition

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d = 2 – the lower critical dimension – is very special …

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Scaling theory (1979)

Study the change of the generalized dimensionless conductance g with the typical size given by L

d = 2 – the lower critical dimension – is very special ...

- The dynamics is *always* localized
- The localization length scales *exponentially* with the (inverse) disorder strength :

$$\xi \propto l\, e^{(\pi k l/2)}$$

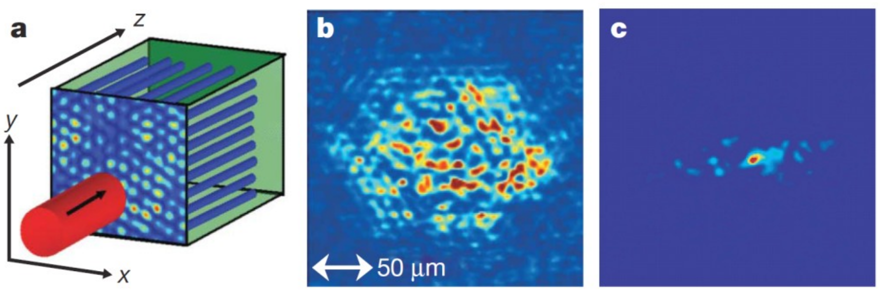
l : mean-free path in the disordered medium
k : wavevector

2 signatures of 2D Anderson localization

E. Abrahams et al, Scaling Theory of Localization: Absence of Quantum Diffusion in Two Dimensions PRL **42** 673 (1979)

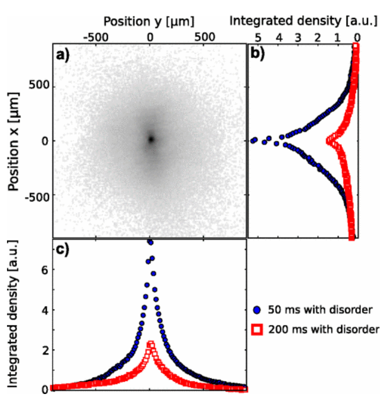
Experiments on 2D disordered systems

Transverse 2D Anderson localization in photonic lattices (Technion)



T. Schwartz *et al.*,
Transport and Anderson localization in disordered 2D photonic lattices,
Nature **446**, 52--55 (2007).

2D Diffusive Expansion of Ultracold Atoms in an Speckle Potential (Institut d'Optique)



M. Robert-de-Saint-Vincent *et al*,
Anisotropic 2D Diffusive Expansion of Ultracold Atoms in a Disordered Potential,
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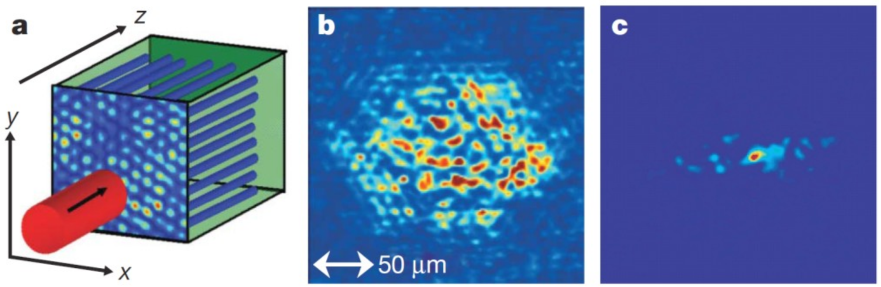

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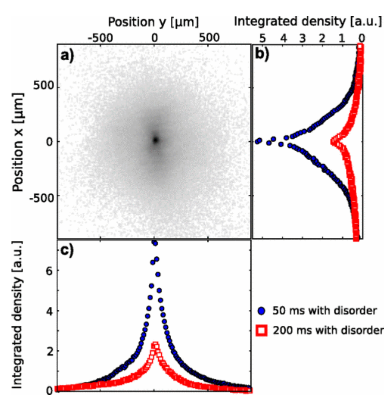

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This work : a quantitative study of 2D Anderson localization

Observation of 2D AL with atomic matter waves
Experimental evidence of the exponential dependance of the localization length with the disorder strength
Arxiv:1504.04987

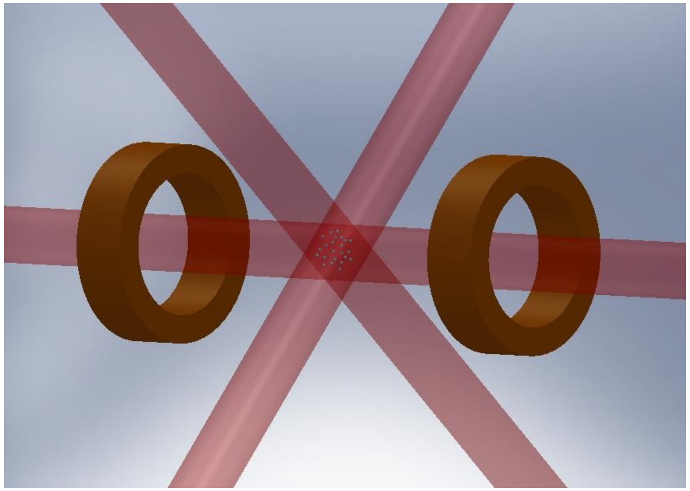
Outline

Anderson localization

Quasiperiodic kicked rotor

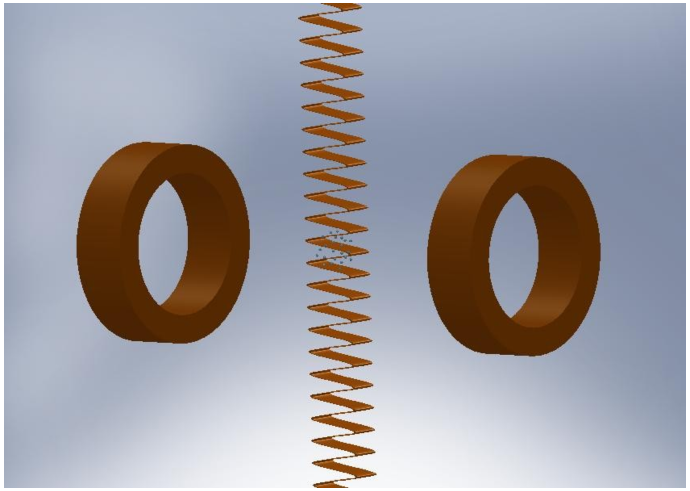
Recent results on weak localization

Periodic Kicked Rotor



Cs Magneto-optical trap

Periodic Kicked Rotor



Pulsed optical lattice

$$H = \frac{p^2}{2} + K \cos x \sum_n \delta(t - n)$$

Adjustable parameters

$\hbar$ reduced Planck constant (*kick frequency*)

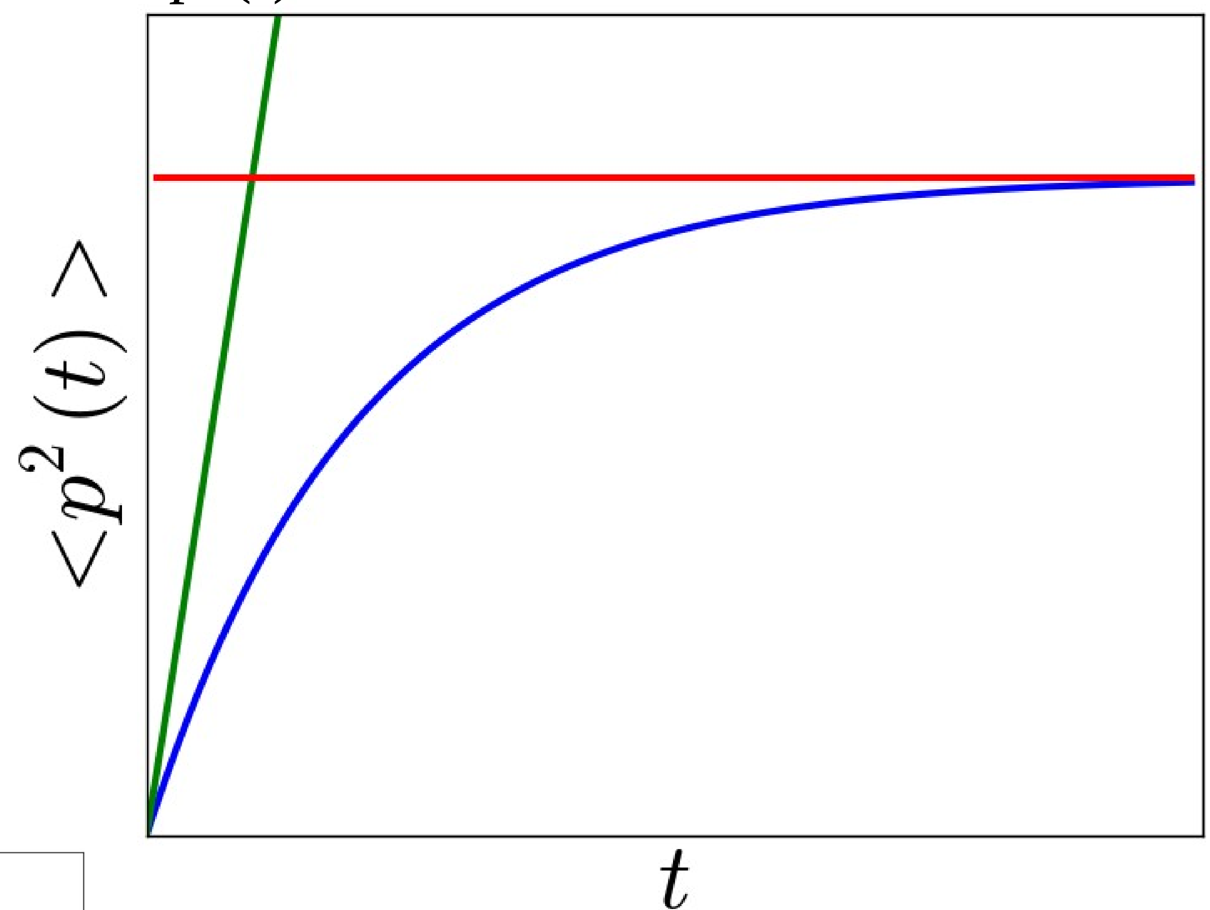
K stochasticity parameter (*laser beam intensity*)

0 to 150 kicks

Dynamical localization

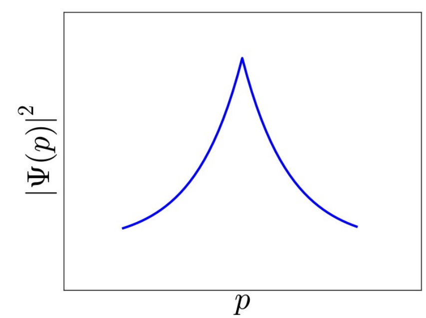
Classic

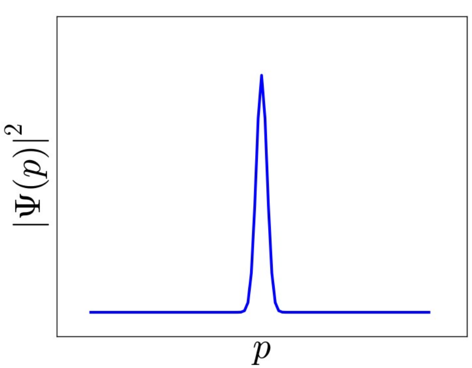
$$< p^2(t) > \propto Dt$$



Quantum

$$< p^2(t) > \propto {p_{loc}}^2$$





Dynamical and Anderson localizations

PHYSICAL REVIEW LETTERS

VOLUME 49 23 AUGUST 1982 NUMBER 8

Chaos, Quantum Recurrences, and Anderson Localization

Shmuel Fishman, D. R. Grempel, and R. E. Prange
Department of Physics and Center for Theoretical Physics, University of Maryland, College Park, Maryland 20742
(Received 6 April 1982)

A periodically kicked quantum rotator is related to the Anderson problem of conduction in a one-dimensional disordered lattice. Classically the second model is always chaotic, while the first is chaotic for some values of the parameters. With use of the Anderson-model result that all states are localized, it is concluded that the *local* quasienergy spectrum of the rotator problem is discrete and that its wave function is almost periodic in time. This allows one to understand on physical grounds some numerical results recently obtained in the context of the rotator problem.

Equivalence between 1D Anderson model and Kicked Rotor

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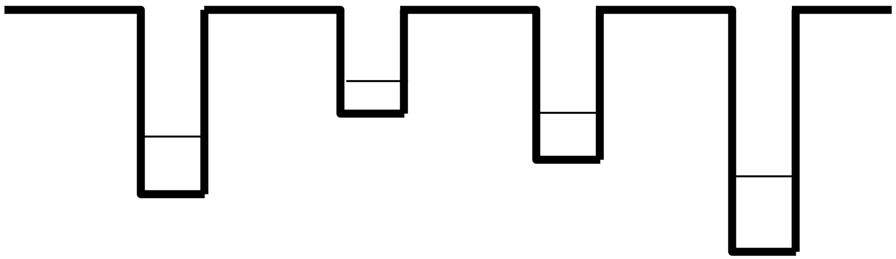
Equivalence between 1D Anderson model and Kicked Rotor

See also (3D case): G. Casati, I. Guarneri and D.L. Shepelyansky, Anderson Transition in a One-Dimensional System with Three Incommensurate Frequencies, Phys. Rev. Lett. 62, 345 (1989)

Dynamical and Anderson localizations

Tight-binding model + disorder



$$H = \sum_n V_n |n\rangle\langle n| + \sum_{n\neq m} t_{nm} |n\rangle\langle m|$$

Disorder
Pseudorandom distribution
(Lorentzian distribution)

$$V_n = \tan\left[\frac{1}{2}(\omega - \hbar\!\!\!k\frac{n^2}{2})\right]$$

Hopping
between site n and site m
(short-range model)

$$t = \tan\left[\frac{K}{2\hbar\!\!\!k}\cos x\right]$$

Dynamical and Anderson localizations

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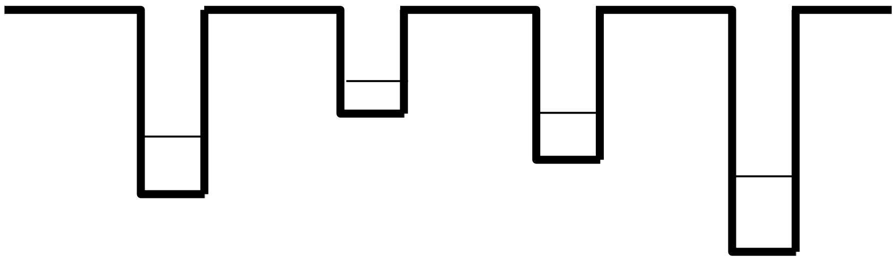

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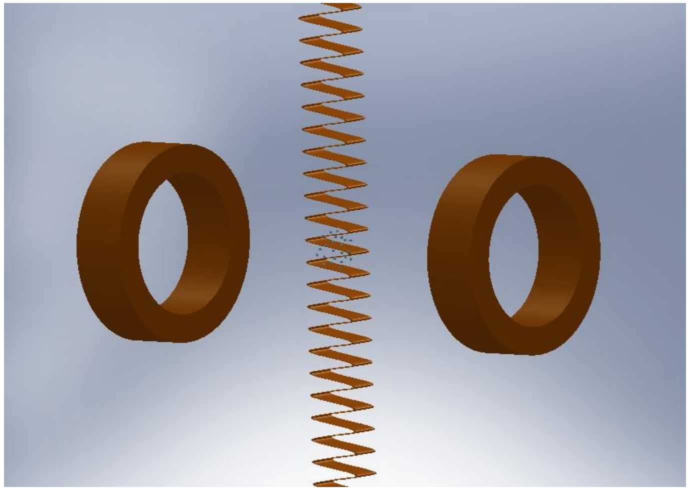
Dynamical localization in the periodic kicked rotor
=
Anderson localization in 1D disordered systems

Quasiperiodic Kicked Rotor



Pulsed optical lattice

$$H_{2D} = \frac{p^2}{2} + K\cos x(1 + \varepsilon cos(\omega_2 t))\sum_n \delta(t-n)$$

Adjustable parameters

$\not{k}$ reduced Planck constant (kick frequency)

K stochasticity parameter (laser beam intensity)

$\pi, \not{k}, \omega_2$ incommensurate triplet

Mapping to the 2D Anderson model

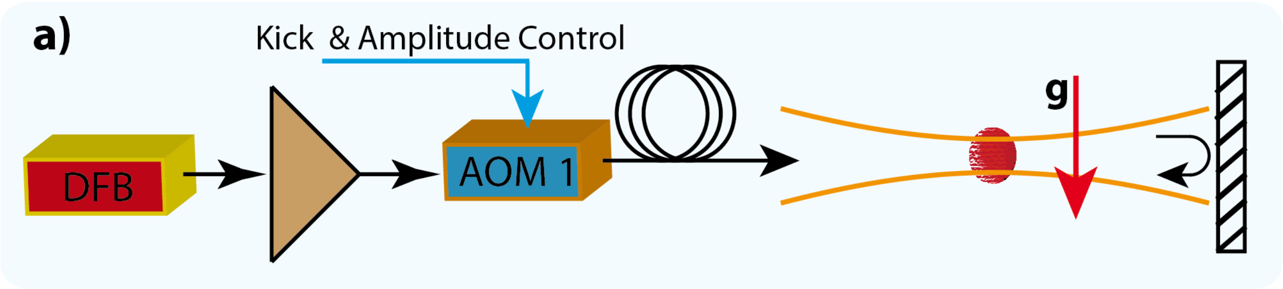
Disorder
Pseudorandom distribution
(Lorentzian distribution)

$$V_{n_1,n_2} = \tan\left[\frac{1}{2}(\omega - \not{k}\frac{n_1^2}{2} + \omega_2 n_2)\right]$$

Hopping
between site n and site m
(short-range model)
Anisotropic for small ε

$$t = \tan\left[\frac{K}{2\not{k}}\cos x_1(1 + \varepsilon\cos x_2)\right]$$

Experimental setup

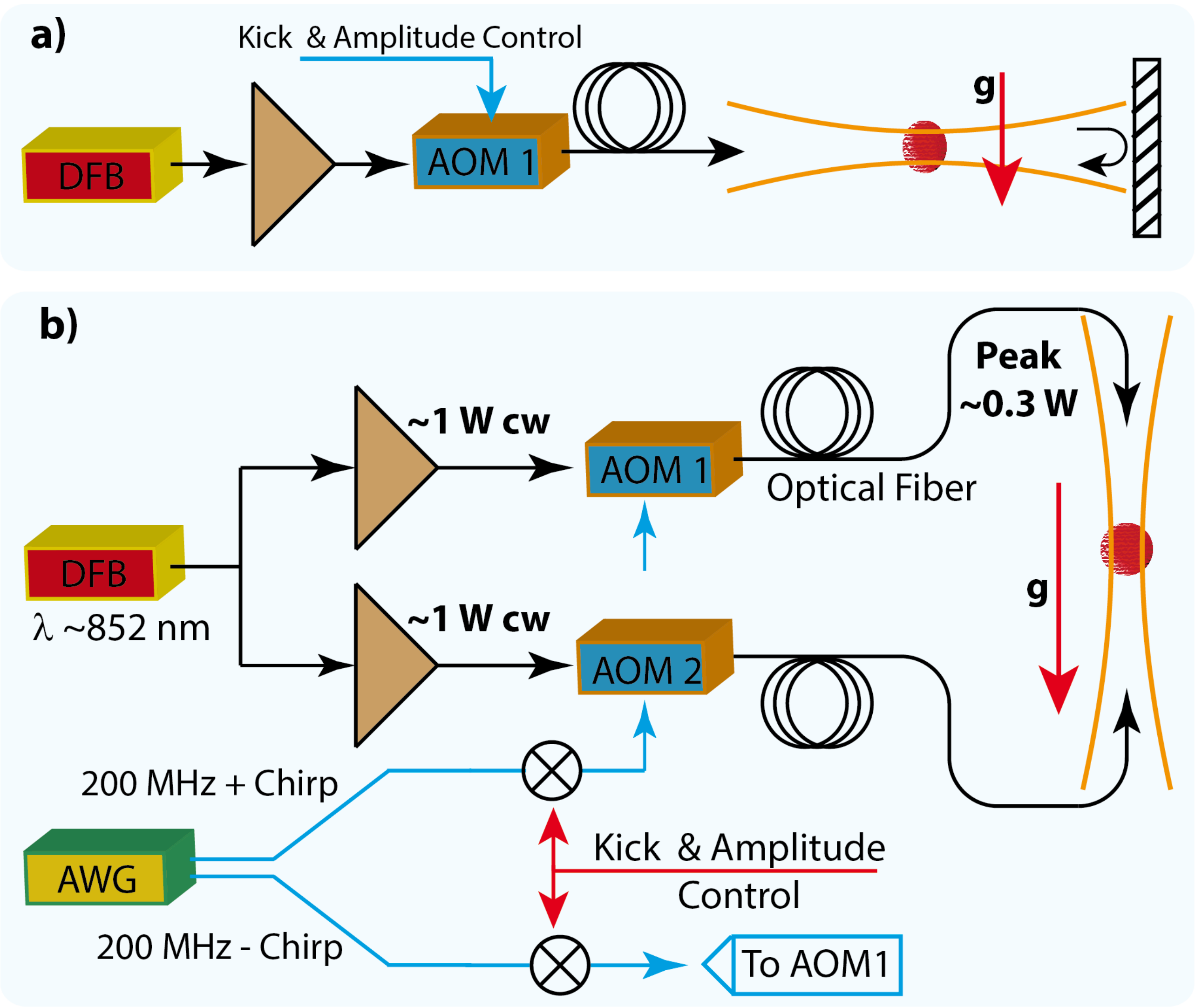


150 kicks

✔ 1D Anderson

✗ 2D Anderson

Experimental setup



→ 150 kicks

✔ 1D Anderson

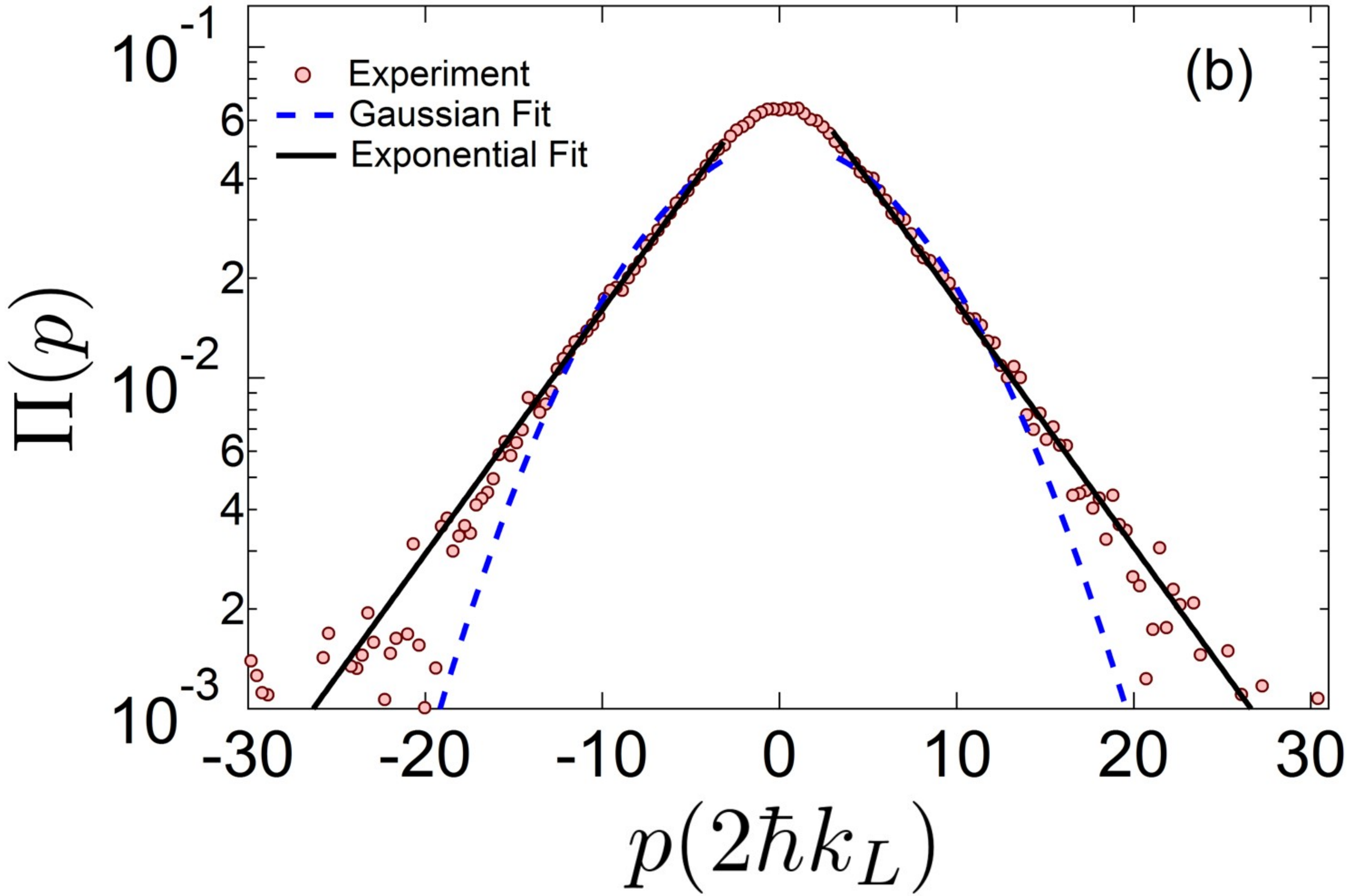
✗ 2D Anderson

→ 1000 kicks

✔ 2D Anderson

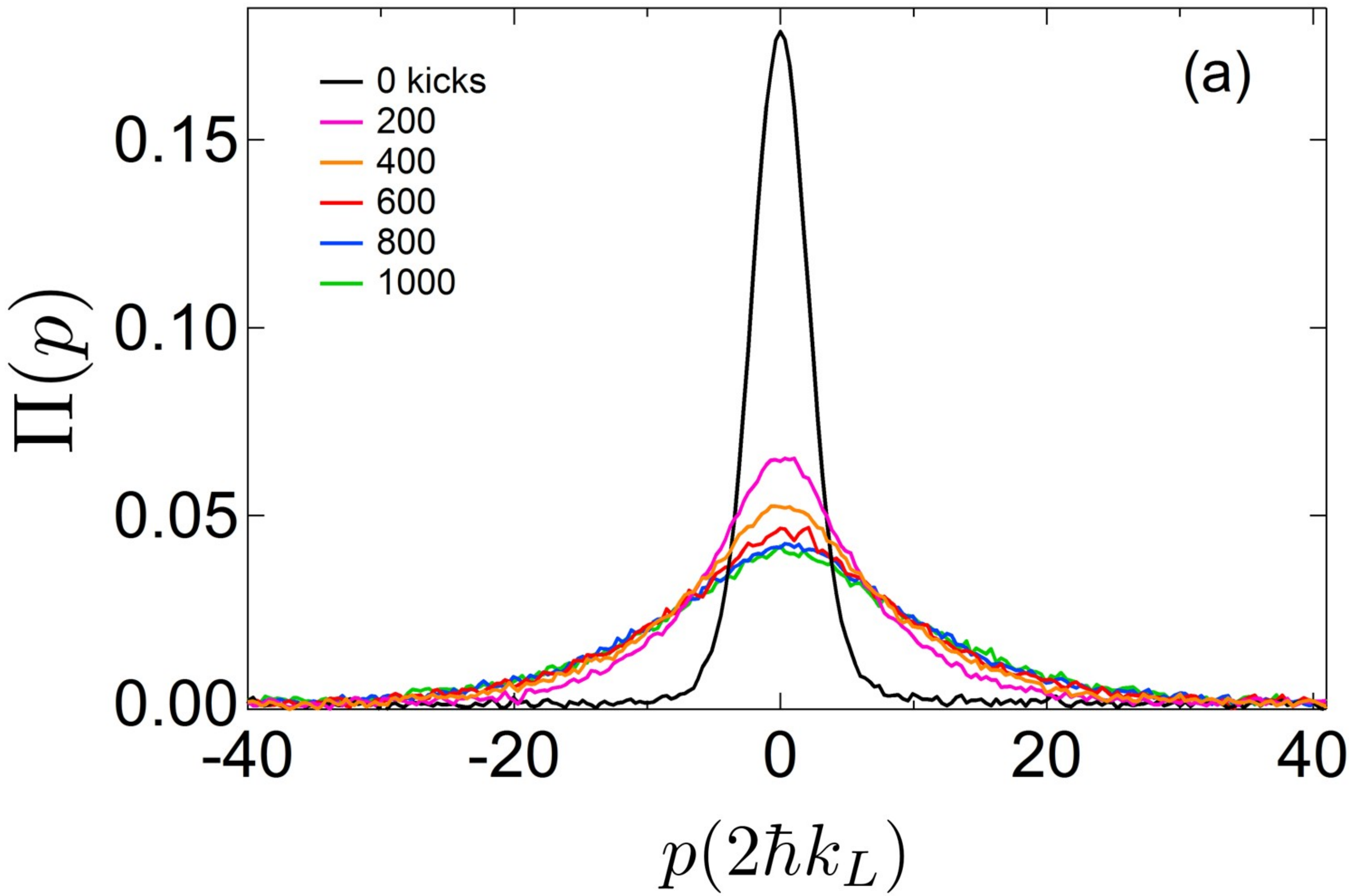
2D Anderson localization

Experimental signature : exponential shape of the momentum distribution at 200 kicks



2D Anderson localization

Experimental signature : freezing dynamics



2D Anderson localization

Self-consistent theory of localization 1D

$$p_{\mathrm{loc}} = \frac{K^2}{4\hbar}$$

D. Vollhardt and P. Wölfle, Phys. Rev. Lett. **48,** 699–702 (1982)

2D Anderson localization

Self-consistent theory of localization 2D

$$p_{\mathrm{loc}} = \frac{K^2}{4\bar{k}} \exp\left(\frac{\alpha\varepsilon K^2}{\bar{k}^2}\right)$$

2D Anderson localization

Self-consistent theory of localization

$$p_{\mathrm{loc}} = \frac{K^2}{4\hbar} \exp\left(\frac{\alpha\varepsilon K^2}{\hbar^2}\right)$$

2D Anderson localization

Self-consistent theory of localization

$$p_{\mathrm{loc}} = \frac{K^2}{4\bar{k}} \exp\left(\frac{\alpha\varepsilon K^2}{\bar{k}^2}\right)$$

$$E_{\mathrm{kin}} \propto \frac{1}{4\Pi_0^2(t)}$$

Population of the zero velocity class

2D Anderson localization

localized momentum distributions after 1000 kicks as a function of the anisotropy parameter

Self-consistent theory of localization

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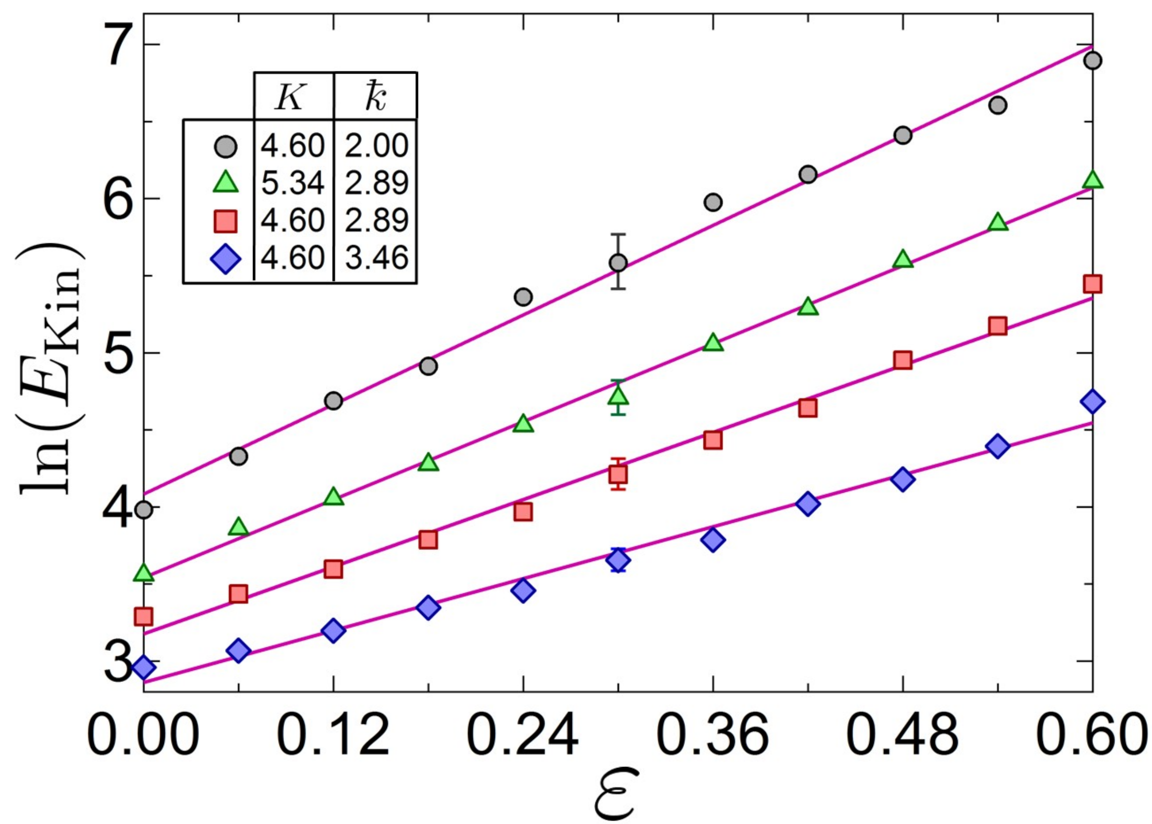

2D Anderson localization

Kinetic energy at 1000 kicks with respect to the purely 1D case vs. the scaling parameter

Self-consistent theory of localization 2D

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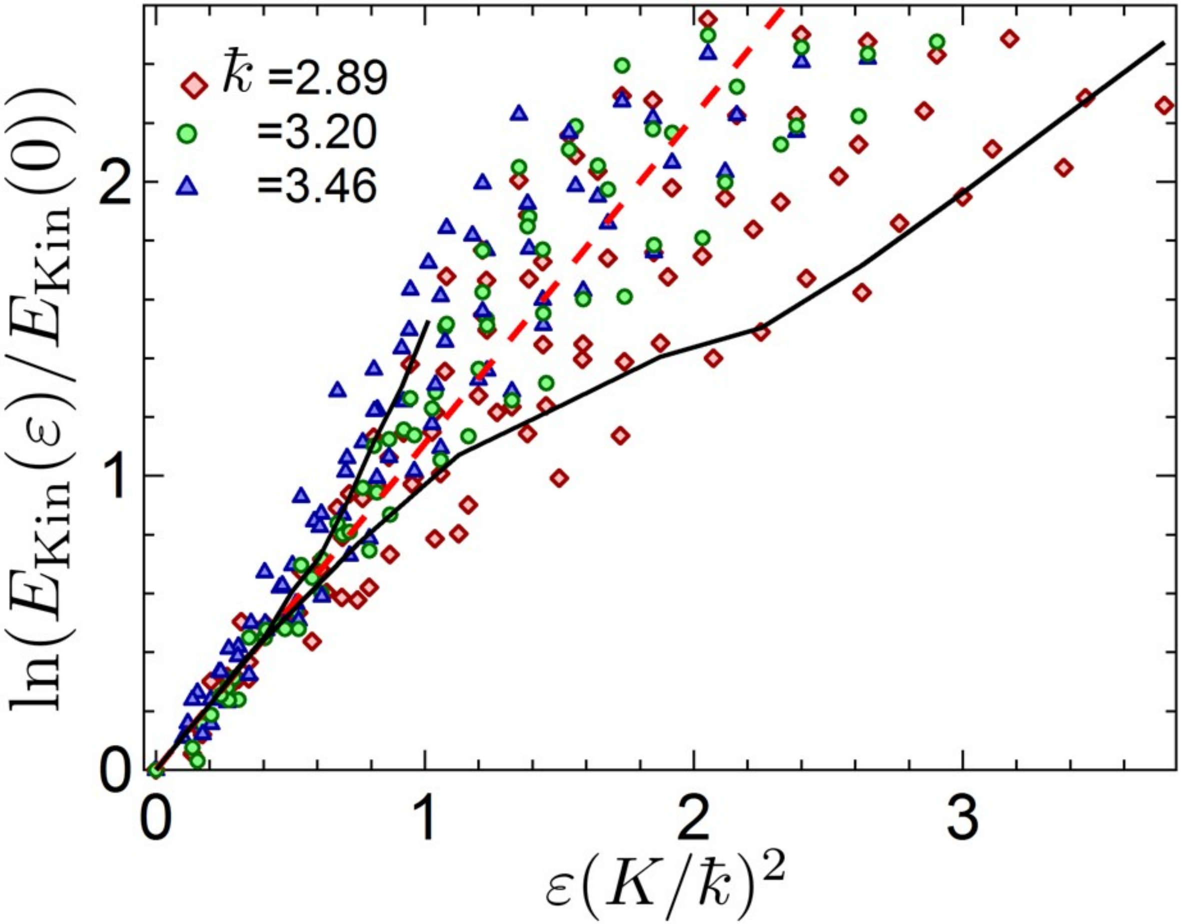

275 measurements for different values of $K, \hbar, \varepsilon$

2D Anderson localization

Kinetic energy at 1000 kicks with respect to the purely 1D case vs. the scaling parameter

Self-consistent theory of localization 2D

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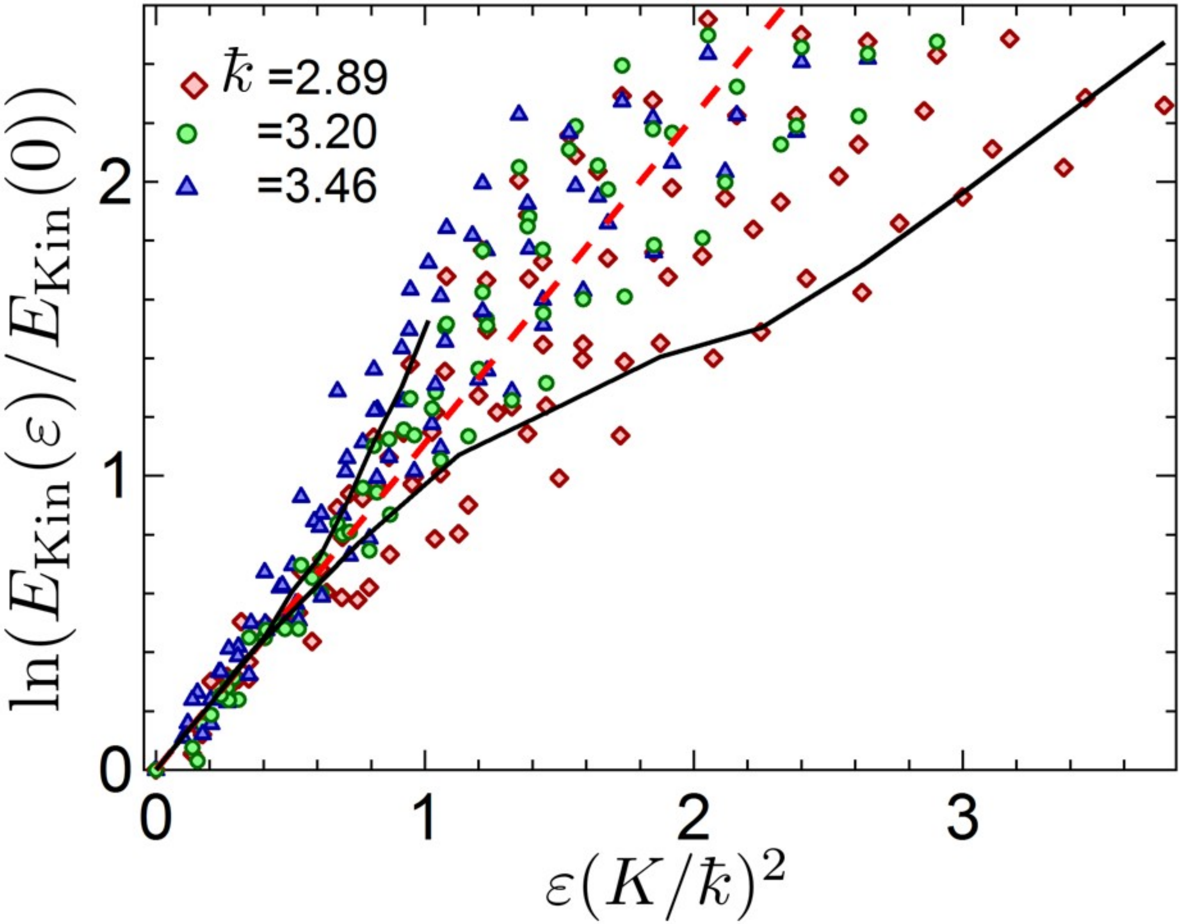

- At the largest K/kbar values, the localization time is not much shorter than 1000 kicks
- The prediction is valid only in the small ε limit and deviations are expected, and indeed observed, at large global parameter
- The K-dependence of the diffusion constant is not perfectly quadratic (oscillatory terms) → *kicked rotor model*

Outline

Anderson localization

Quasiperiodic kicked rotor

Recent results on weak localization

Weak localization

- Precursor of Anderson (strong) localization
- The diffusion coefficient tends to be lowered in presence of disorder

  ➔ Enhanced amplitude probability that the wavepacket comes back to its origin

  ➔ One-constructive-interference effect in time-reversal invariant systems

$$P(r,r') = \sum_i |A_i|^2 + \sum_{i \neq j} A_i A_j^*$$

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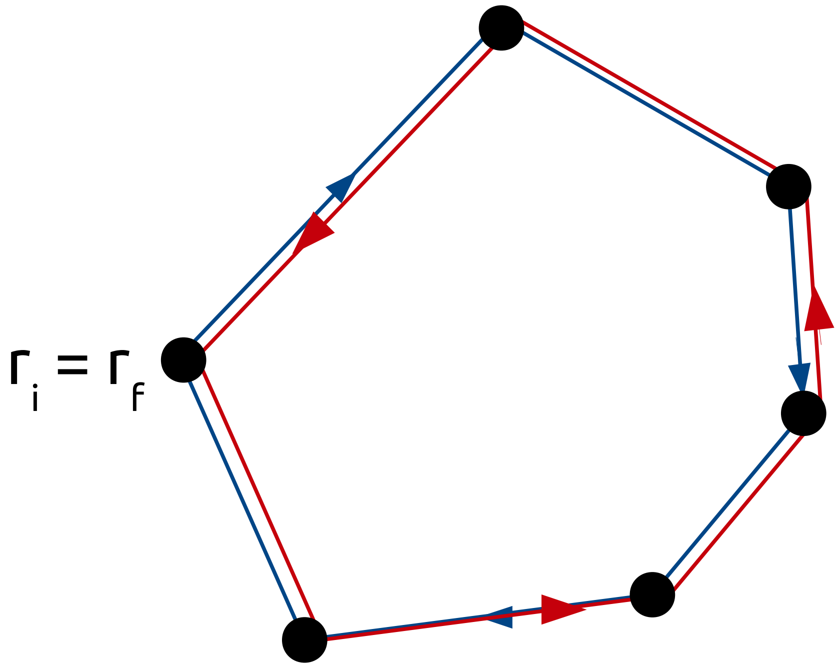

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$$P_{quant}(r,r) = 2\sum_i |A_i|^2$$

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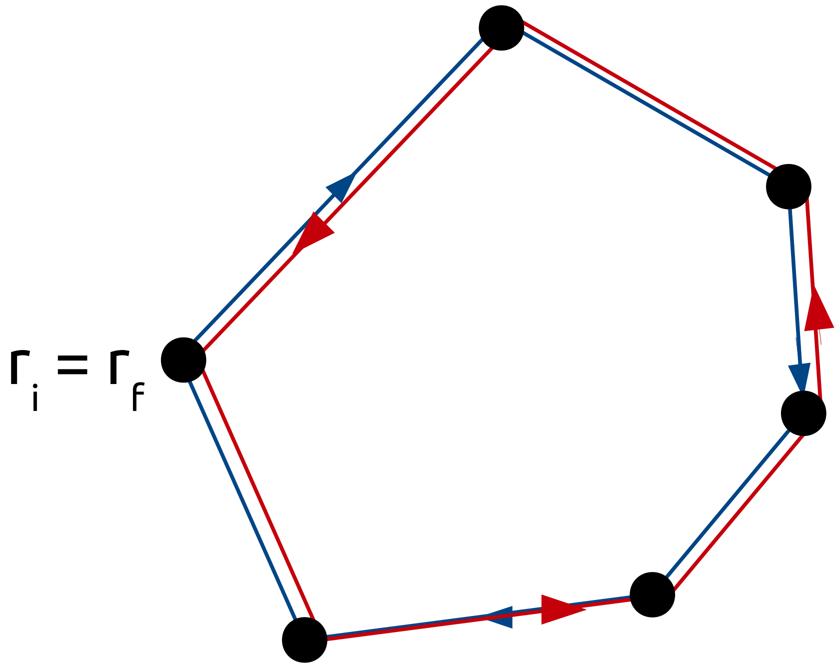

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Enhanced Return to the Origin

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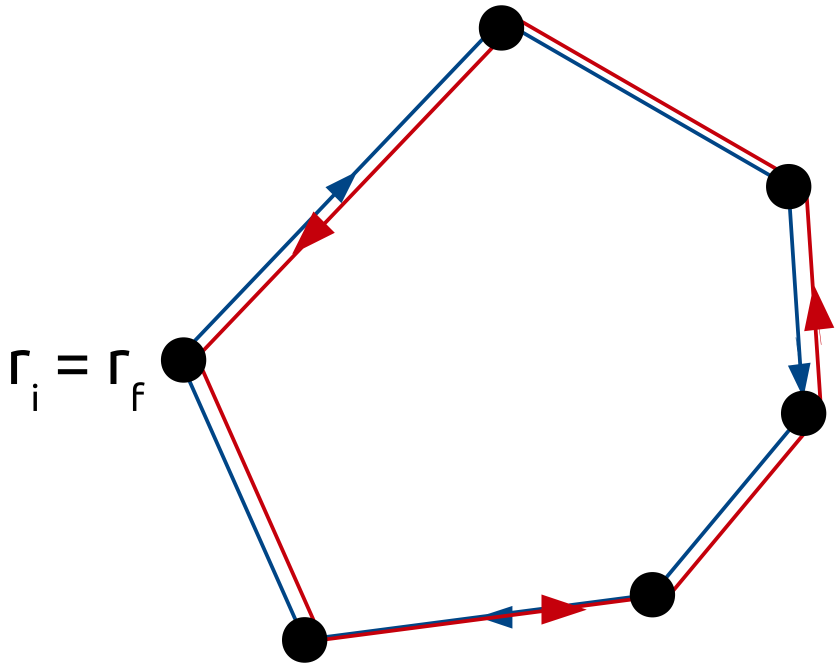

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Enhanced Return to the Origin

See also:
(transmission exp.)

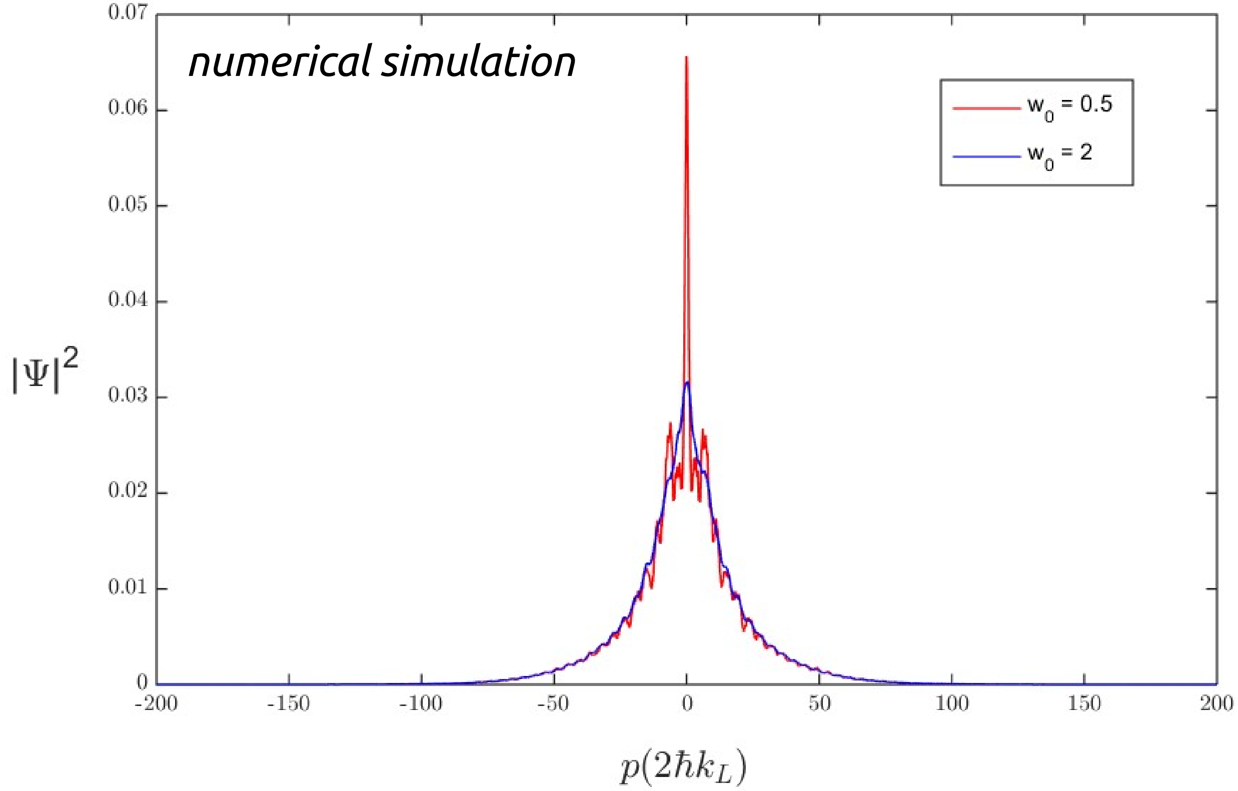
Coherent Backscattering of Ultracold Atoms,

F. Jendrzejewski, K. Müller, J. Richard, A. Date, T. Plisson, P. Bouyer, A. Aspect, and V. Josse

Phys. Rev. Lett. **109**, 195302 (2012)

Weak localization in the kicked rotor

✗ Extreme sensitivity to initial width of the momentum distribution



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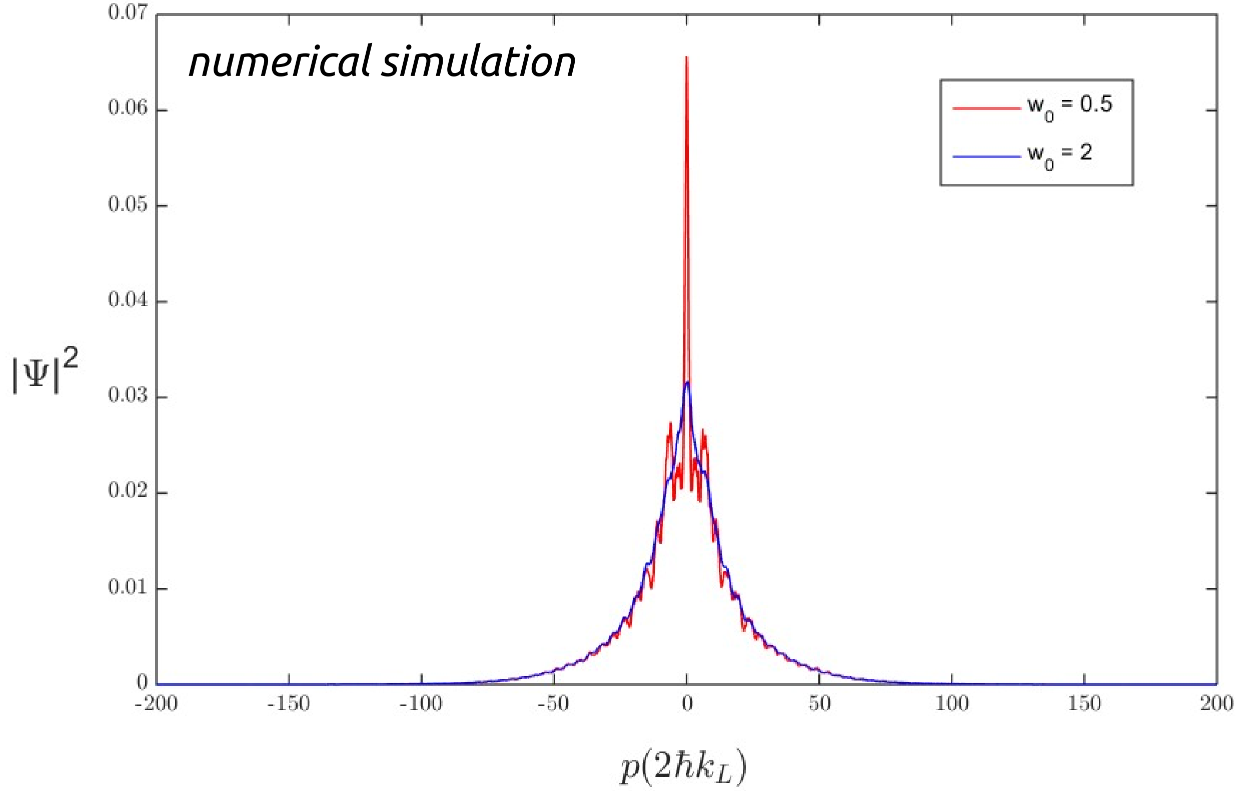

✗ The peak is always here !

✔ Control of the number of scattering events

Weak localization in the kicked rotor

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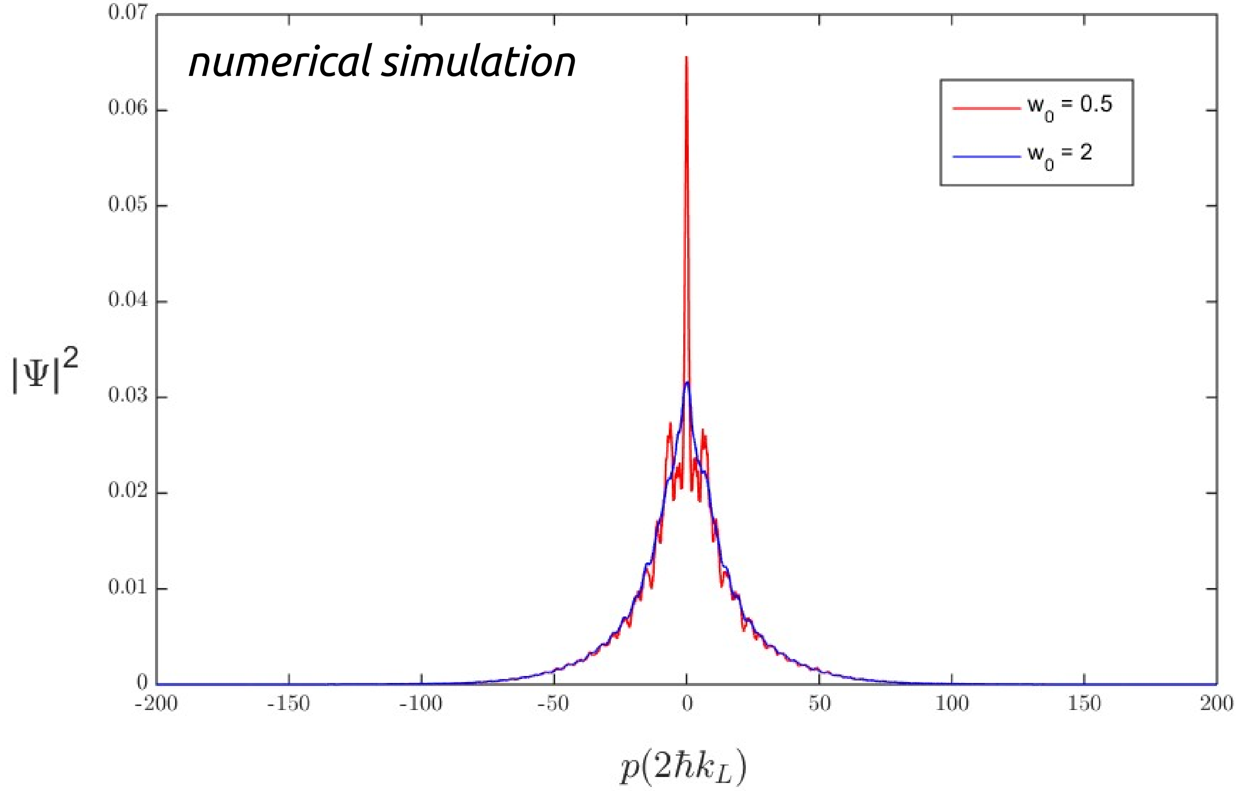

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Perform a differential measurement
(i.e. breaking sometimes this interference effect)

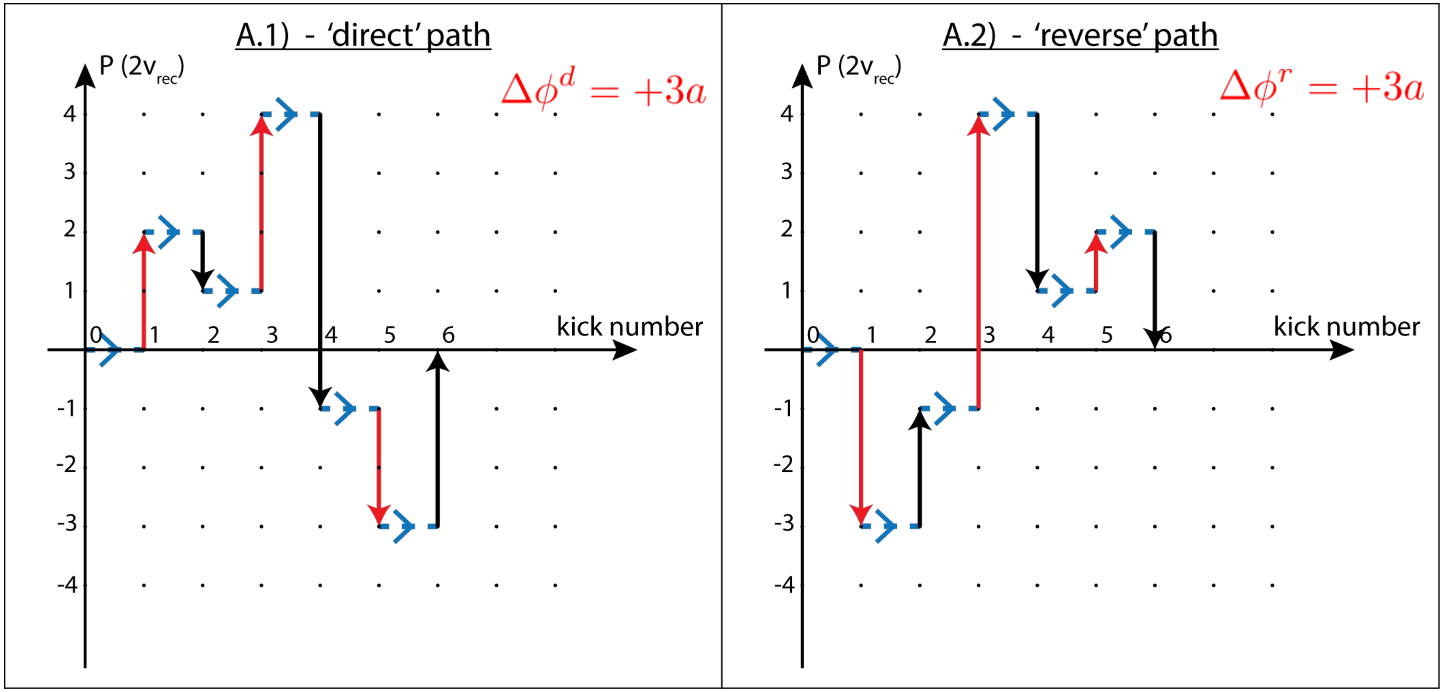
Weak localization in the kicked rotor

Perform a differential measurement
(i.e. breaking sometimes this interference effect)

Periodically-shifted kicked rotor

$$H = \frac{p^2}{2} + K\cos(x)\sum_n \delta(t-2n) + K\cos(x+a)\sum_n \delta(t-2n+1)$$

Even kick number: $\Delta\phi^d = +\Delta\phi^r$ *Same additional phase*



C. Tian, A. Kamenev, and A. Larkin, Phys. Rev. B **72**, 045108 (2005)

Weak localization in the kicked rotor

Perform a differential measurement
(i.e. breaking sometimes this interference effect)

Periodically-shifted kicked rotor

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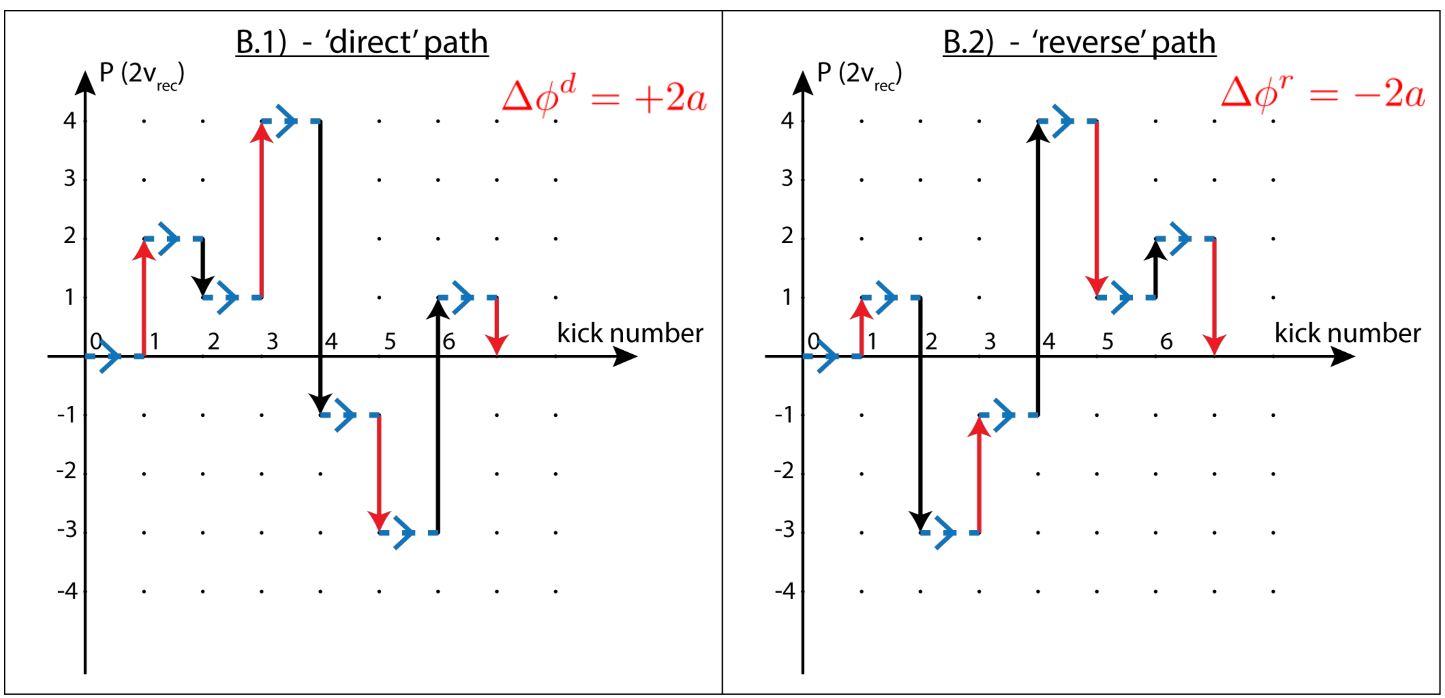
Odd kick number: $\Delta\phi^d = -\Delta\phi^r$ *Opposite signs*



Weak localization in the kicked rotor

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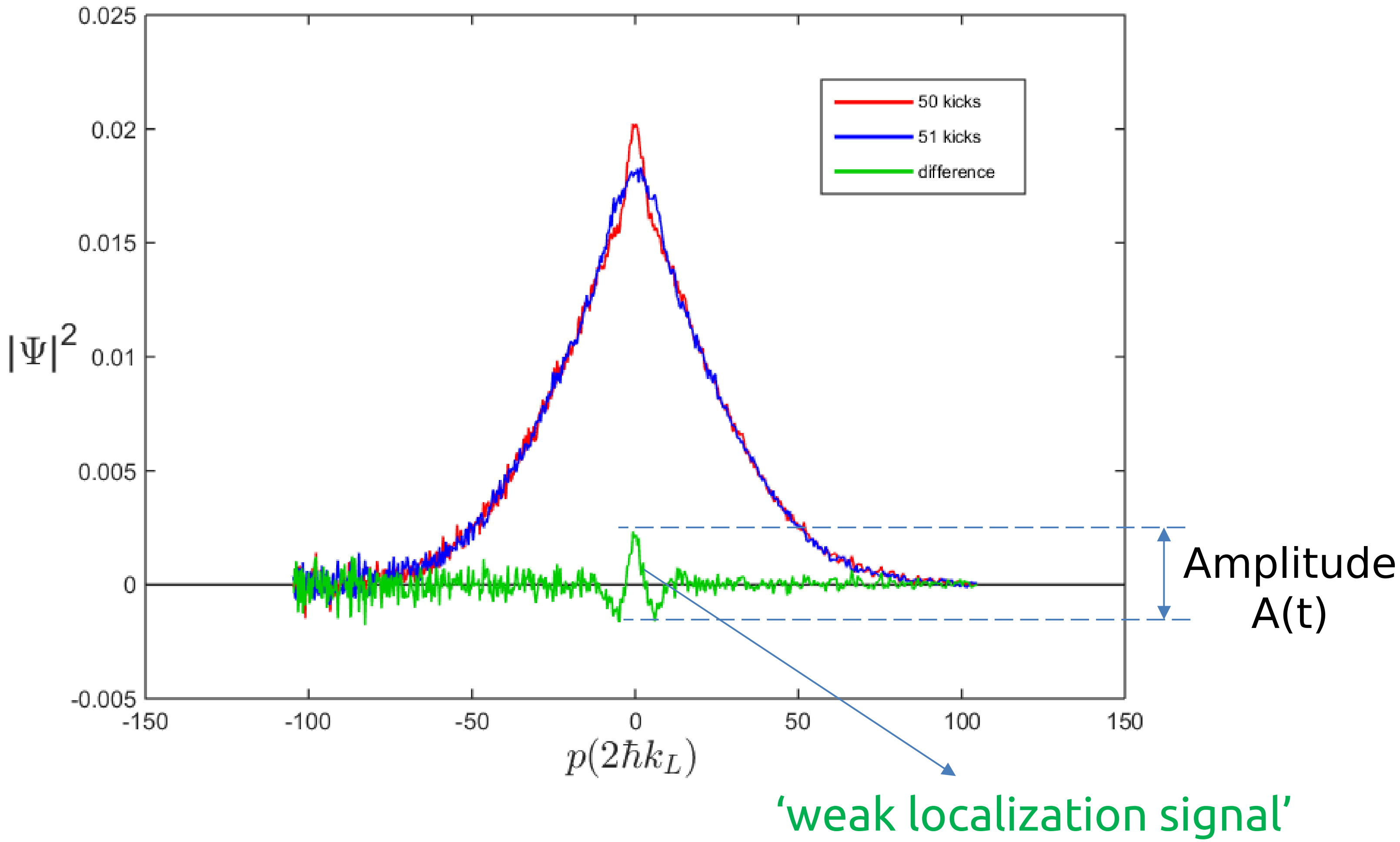
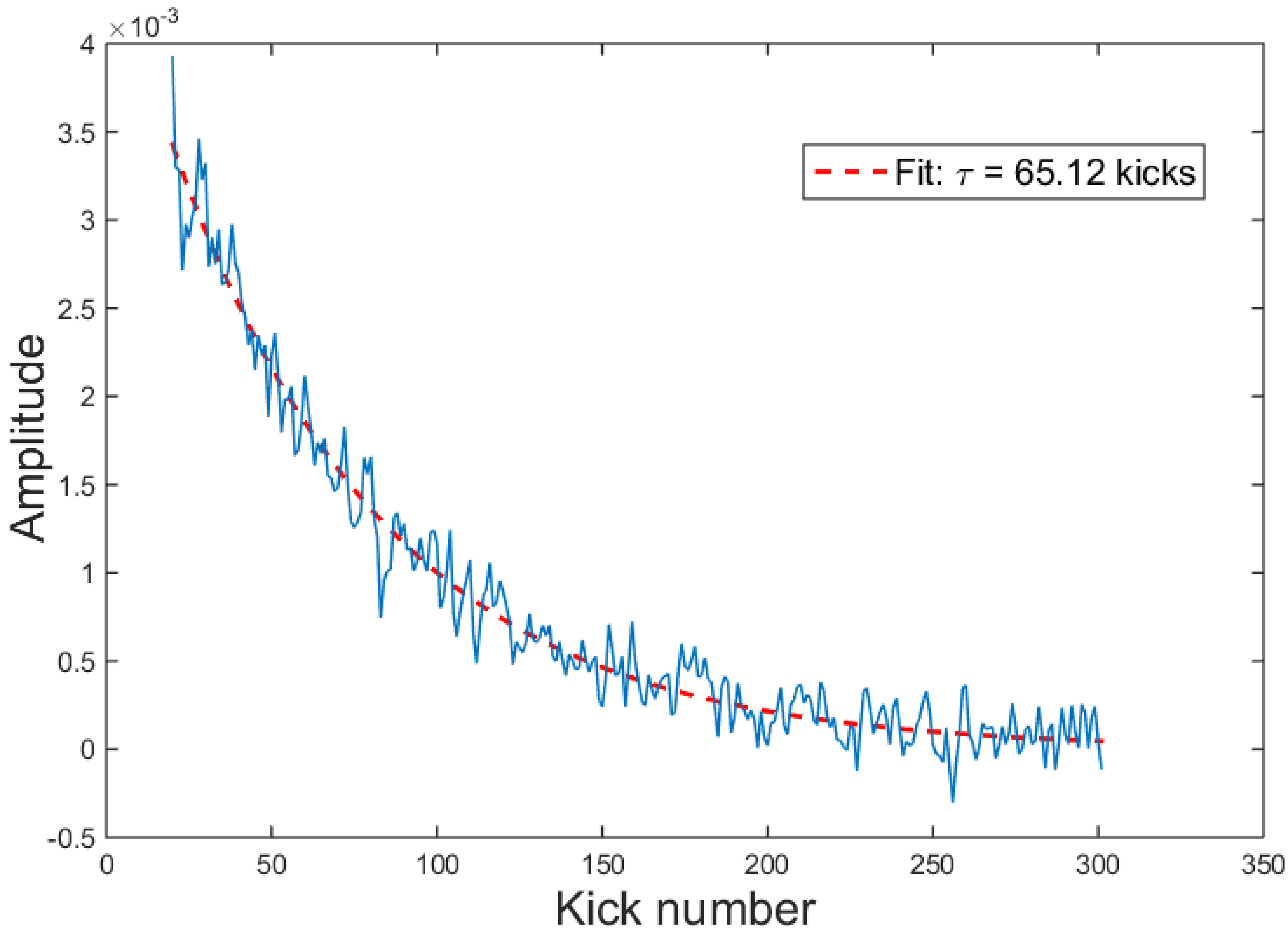


Weak localization in the kicked rotor

A useful tool to probe decoherence of our system



Conclusion

Cs experiment

- A quantitative study of 2D Anderson localization with the quasiperiodic kicked rotor :
  → ***First experimental evidence of 2D Anderson localization with atomic matter waves***
  → ***experimental evidence that d=2 is the lower critical dimension***

- Observation of the weak localization with the periodic kicked rotor

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- *A quasiperiodic kicked rotor with 4 incommensurate frequencies ↔ 4D Anderson model (Transition)*

- *A quasiperiodic kicked rotor with 3 incommensurate frequencies in others symmetry classes*

- *Anderson model with interactions*

^{39}K experiment (under construction)

PhLAM (Lille)
Denis Bacquet
Radu Chicireanu
Jean-Claude Garreau
Clément Hainaut (PhD since 2014)
Jamal Khalloufi (PhD since 2014)
Isam Manaï (Post-doc since 2014)
Pascal Szriftgiser
Samir Zemmouri (PhD since 2015)
Véronique Zehnlé
J-F Clément
Matthias Lopez (PhD 2009-2012, now at SYRTE)
Benoît Vermersch (PhD 2010-2013, now at IQOQI Innsbruck)

Laboratoire Kastler-Brossel (Paris)
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Laboratoire de Physique Théorique (Toulouse)
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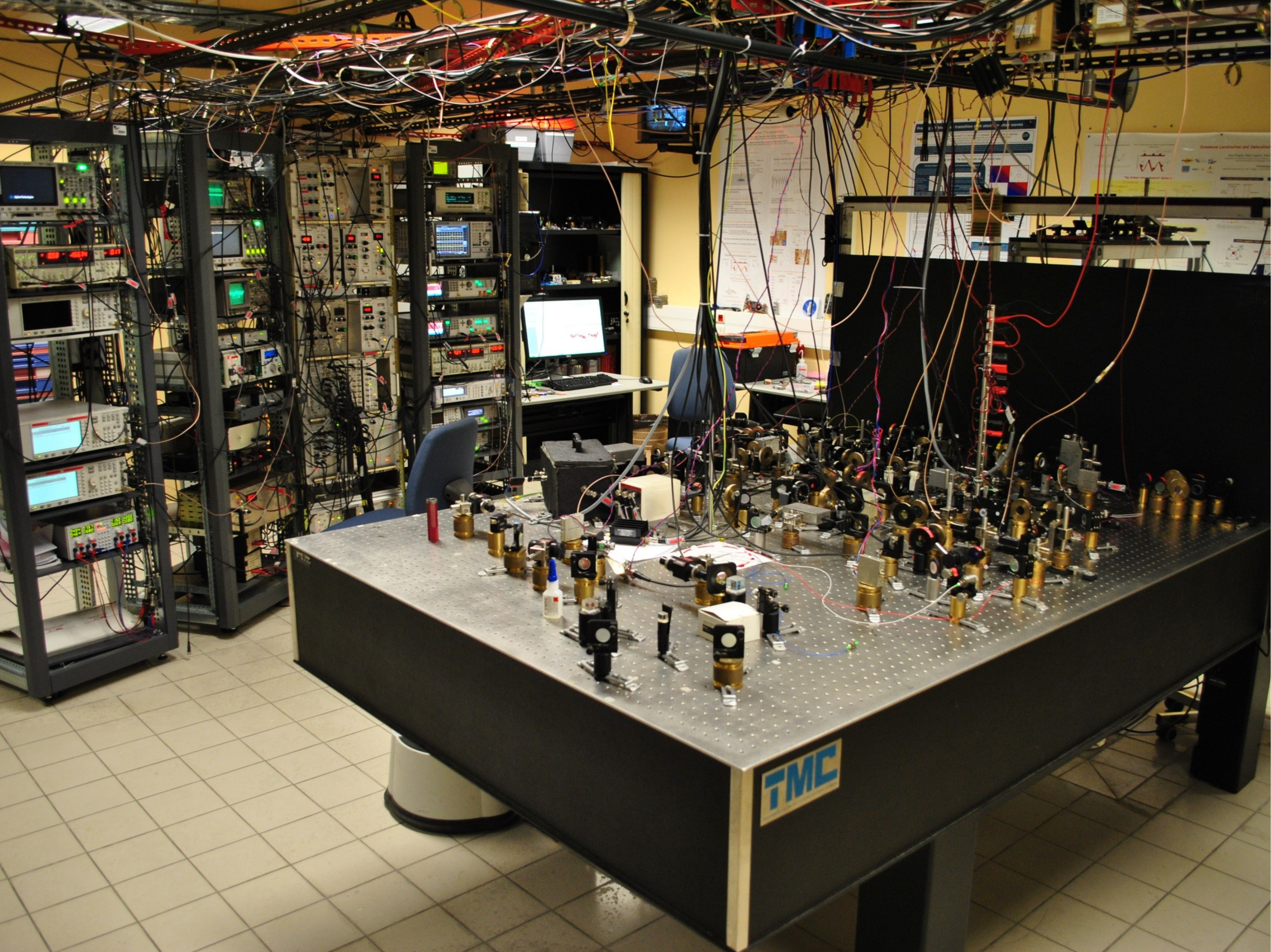
Laboratoire Painlevé (Lille)
Stephan De Bièvre


TMC