Spontaneous creation and dynamics of vortices in Bose-Einstein condensates

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Journées GdR Atomes Froids et IFRAF
École Normale Supérieure, Paris
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• Introduction to the Kibble-Zurek mechanism
• Creating defects in Bose condensates via the Kibble-Zurek mechanism
• Defect’s characterization
• Dynamics & interactions
the Kibble-Zurek mechanism

- Second-order phase transitions
- Finite rate crossing
- Spontaneous and stochastic production of defects
the Kibble-Zurek mechanism

Power-law scaling

\[ \xi(t) = \frac{\xi_0}{|\epsilon(t)|^\nu} \]

\[ \tau(t) = \frac{\tau_0}{|\epsilon(t)|^{z\nu}} \]

reduced parameter:

\[ \epsilon = \frac{\lambda_c - \lambda}{\lambda_c} \]

Case of linear quench

\[ \epsilon(t) = \frac{t}{\tau_Q} \]

time to the transition

\[ \tau(t) \approx \frac{\epsilon}{\dot{\epsilon}} \]

“freezing time”

\[ \hat{t} \sim \left( \tau_0 \tau_Q^{z\nu} \right)^{\frac{1}{1+z\nu}} \]

domain size

\[ \hat{\xi} = \xi(\hat{t}) = \xi_0 \left( \frac{\tau_Q}{\tau_0} \right)^{\frac{\nu}{1+z\nu}} \]

density of defects

\[ d \sim \hat{\xi}^{-D} \]

Liquid crystals: isotropic/nematic

C. Bauerle et al. (1996)
V.M.H Ruutu et al. (1996)

R. Monaco et al. (2009)

S. Ulm et al. (2013)
K. Pyka et al. (2013)

Bose gases: ferromagnetic

L. E. Sadler et al. (2006)

C.N. Weiler et al. (2008)

D. Chen et al. (2011)
S. Braun et al. (2014)

N. Navon et al. (2015)

Liquid $^3$He: normal/SF

Liquid W

Bose gases: thermal/BEC, D < 3

Hom. Bose gases: thermal/BEC

1D ion crystals: linear/zig-zag

Breaking the symmetry

Formation of defects

Bose gases: thermal/BEC

L. Corman et al. (2014)
experimental setup in Trento

G. Lamporesi et al.,
ToF expansion of a BEC

expansion time limited to ~ 40 ms due to the gravity fall

magnetic levitation against the gravity to increase the expansion time
Key observation: the number of defects strongly depends on the rate at which the BEC transition is crossed !!
Generating solitons via the Kibble-Zurek mechanism

W. H. Zurek, PRL 102, 105702 (2009)
Check: change the cooling/quench time.
**guess:** these are gray solitons spontaneously nucleated at the BEC transition by the Kibble-Zurek mechanism (KZM)!!

imaging resolution: 10 \( \mu \text{m} \)
soliton width in trap: \( \xi(0) = 200-250 \text{ nm} \)
width after TOF: \( \xi(180 \text{ ms}) = 50-100 \text{ \( \mu \text{m} \)} \)

G.Lamporesi *et al.*, Nat. Phys. 9, 656 (2013)
the number of defects is expected to follow a power-law as a function of the quench time (fixed size of the system)

\[ N_s \mu \tau_Q^{-\alpha} \]

where \( \alpha \) is determined by the critical exponents of the phase transition.

W. H. Zurek
PRL 102, 105702 (2009)

**OK**, we can count our solitons!
Measurement of the KZ $\alpha$ coefficient

\[ N_s \mu \tau_Q^{-\alpha} \]

$\alpha = 1.4$

to compare with the available theoretical prediction (Zurek 2009)

1D, homogeneous temperature

$\alpha = 7/6 \sim 1.17$

G.Lamporesi et al., Nat. Phys. 9, 656 (2013)
**The lifetime puzzle**

![Graph showing the average number of solitons over time](image)

- **Solitons** are expected to be **unstable**
  - Thermally (unless at T=0)
  - Dynamically (due to snake instabilities)

\[
\gamma = \frac{\mu}{\hbar \omega_\perp} = \frac{R_\perp}{2 \xi}
\]

... and to decay into **vortex rings**


(Also in DFG at MIT)
Solitonic vortices

Vortex oriented perpendicularly to the axis of an axisymmetric elongated trap.

- Quantized vorticity
- Anisotropic phase pattern
- Planar density depletion

\[ \gamma = \frac{\mu}{\hbar \omega_\perp} = \frac{R_\perp}{2\xi} \]

Brand et al., JPB 34, L113 (2001)

Komineas et al., PRA 68, 043617 (2003)

Brand et al., PRA 65, 043612 (2002)
Solitonic vortices

Density in trap

Phase

Density after free expansion

Asymmetric twist

(a) VORTEX

(b) SOLITON VORTEX in a cigar-shaped trap

(c) SOLITON in a cigar-shaped trap

M. Tylutki et al., EPJ-ST 224, 577 (2015)
Solitonic vortices
Random orientation

Triaxial absorption imaging after long TOF

Random number
Random circulation

homodyne detection of the phase pattern by interfering two copies of the condensate

S. Donadello et al., PRL 113, 065302 (2014)
Scaling exponents

\[ n \sim \frac{\xi_d}{\xi_D} = \frac{1}{\xi_0 (D-d)} \left( \frac{\tau_0}{\tau_Q} \right)^{(D-d)\frac{\nu}{1+\nu}} \]

\( \nu, z \): critical exponents
D: system dimension
d: defect dimension

\((D-d)=2\)

\((D-d)=1\)

\[ (\alpha=1.4) \]

<table>
<thead>
<tr>
<th>Critical exponents \ Trap</th>
<th>Homogeneous</th>
<th>Harmonic</th>
<th>Toroidal</th>
</tr>
</thead>
<tbody>
<tr>
<td>Arbitrary ((\nu, z))</td>
<td>(\frac{2\nu}{1+\nu z})</td>
<td>(\frac{2(1+2\nu)}{1+\nu z})</td>
<td>(\frac{1+3\nu}{1+\nu z})</td>
</tr>
<tr>
<td>Mean-field theory ((\nu = \frac{1}{2}, z = 2))</td>
<td>(\frac{1}{2})</td>
<td>2</td>
<td>(\frac{5}{4})</td>
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<tr>
<td>Experiments/F model ((\nu = \frac{2}{3}, z = \frac{3}{2}))</td>
<td>(\frac{2}{3})</td>
<td>(\frac{7}{3})</td>
<td>(\frac{3}{2})</td>
</tr>
</tbody>
</table>

Del Campo et al., NJP 13, 083022 (2011)

Zurek, PRL 102, 105702 (2009)
Search for effects on KZ scaling due to geometry of confinement:

- dimensional cross-over
- creation of different types of defects

Revised evaporation ramps suppressing the effects of decay of defects
- power-law scaling for slow ramps
- aspect ratio dependent exponent

- flat plateau for fast ramps
- plateau independent on aspect ratio

<table>
<thead>
<tr>
<th></th>
<th>model</th>
<th>$\alpha_{\text{homog}}$</th>
<th>$\alpha_{\text{harm}}$</th>
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</thead>
<tbody>
<tr>
<td>solitons in 3D or in 1D ($S - D = 1$)</td>
<td>MF</td>
<td>1/4</td>
<td>1</td>
</tr>
<tr>
<td>vortices in 3D ($S - D = 2$)</td>
<td>MF</td>
<td>1/2</td>
<td>2</td>
</tr>
<tr>
<td></td>
<td>F-model</td>
<td>1/3</td>
<td>7/6</td>
</tr>
<tr>
<td></td>
<td>F-model</td>
<td>2/3</td>
<td>7/3</td>
</tr>
</tbody>
</table>
Dynamics of quantized vortices

Determine dissipative and transport processes in:

• Superfluid helium
• Superconductors
• Neutron stars

In atomic BECs:
Controllable environment, spatial scale from $\xi$ to tens of $\xi$, inhomogeneous systems, boundary physics...

BUT

Vortices are produced stochastically and their dynamics cannot be followed through standard destructive absorption imaging
Stroboscopic imaging of defect dynamics

$\mu$-wave pulses extract a small fraction from the BEC

$|1, -1\rangle \rightarrow |2, -2\rangle$

$\frac{\Delta N}{N_0} \sim 4\%$

- Initial atom number $\sim 10^7$
- Magnetic harmonic trap in $|1, -1\rangle$ with $\{\omega_x, \omega_y = \omega_\perp, \omega_z\}/2\pi = \{131, 13\}$ Hz
- 13 ms expansion in $|2, -2\rangle$ plus RF dressing
- Selective imaging of the output coupled fraction

Stroboscopic imaging of defect dynamics

- expansion in the anti-trapped state
- selective imaging of the output coupled fraction
- imaging iterated up to 20 times
- plot residuals from the Thomas-Fermi profile

S. Serafini et al., PRL 115, 170402 (2015)
Stroboscopic imaging of defect dynamics

- expansion in the anti-trapped state
- selective imaging of the output coupled fraction
- imaging iterated up to 20 times
- plot residuals from the Thomas-Fermi profile

S. Serafini et al., PRL 115, 170402 (2015)
A straight vortex line is expected to precess in an inhomogeneous non-rotating condensate, following an equipotential elliptical orbit around the center:

\[ T_{SV} = \frac{4(1 - r_0^2)\mu}{3\hbar \omega_{\perp} \ln(R_{\perp}/\xi)} T_z \]

\[ T_z = \frac{2\pi}{\omega_z} \quad \text{axial trapping period} \]

\[ r_0 = \frac{z_{\text{max}}}{R_z} = \frac{y_{\text{max}}}{R_{\perp}} \quad \text{normalized oscillation amplitude} \]

\[ \xi \quad \text{condensate healing length} \]


Period VS atom number

\[ T_{SV} \propto \mu \propto N(t)^{2/5} \]
Period VS amplitude of orbit

\[ \frac{T_{SV}}{T_z} = \frac{4(1 - r_o^2)\mu}{3\hbar\omega_\perp \ln(R_\perp/\xi)} \]
Interaction among vortices

Random orientation of the nodal lines in the radial plane

Full 3D vortex interaction

Ideal benchmark for:
- Vortex annihilation
- Vortex decay
- Vortex reconnection

Present simulations: Vortices are initially at rest

Our experiment: finite relative momentum

Reconnection in liquid crystals

Chuang et al., Science 251, 1336 (1991)
Interaction among vortices: lifetime measurement

1 or 2 vortices: decay by dissipation with the thermal fraction
3 vortices: faster decay
Interaction among vortices: phase delays

- Frequently: no visible interactions
- Frequently: change of visibility
- Sometimes: phase shifts
- Seldom: annihilations

Single reconnection energetically expensive due to nodal line stretching.

Possible alternatives:
- double reconnection
- rotation of the nodal lines when approaching

Summary

formation

Phase transition
Temperature
Time
Fast
Domain boundary
Slow

dynamics & interaction

nature

future developments

- investigation of post-quench dynamics after crossing phase transitions
- microscopic study of reconnection mechanisms
Thank you!

Simone Serafini
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