

# ***Spontaneous creation and dynamics of vortices in Bose-Einstein condensates***

Gabriele Ferrari

INO-CNR BEC Center, TIFPA-INFN and  
Dipartimento di Fisica, Università di Trento

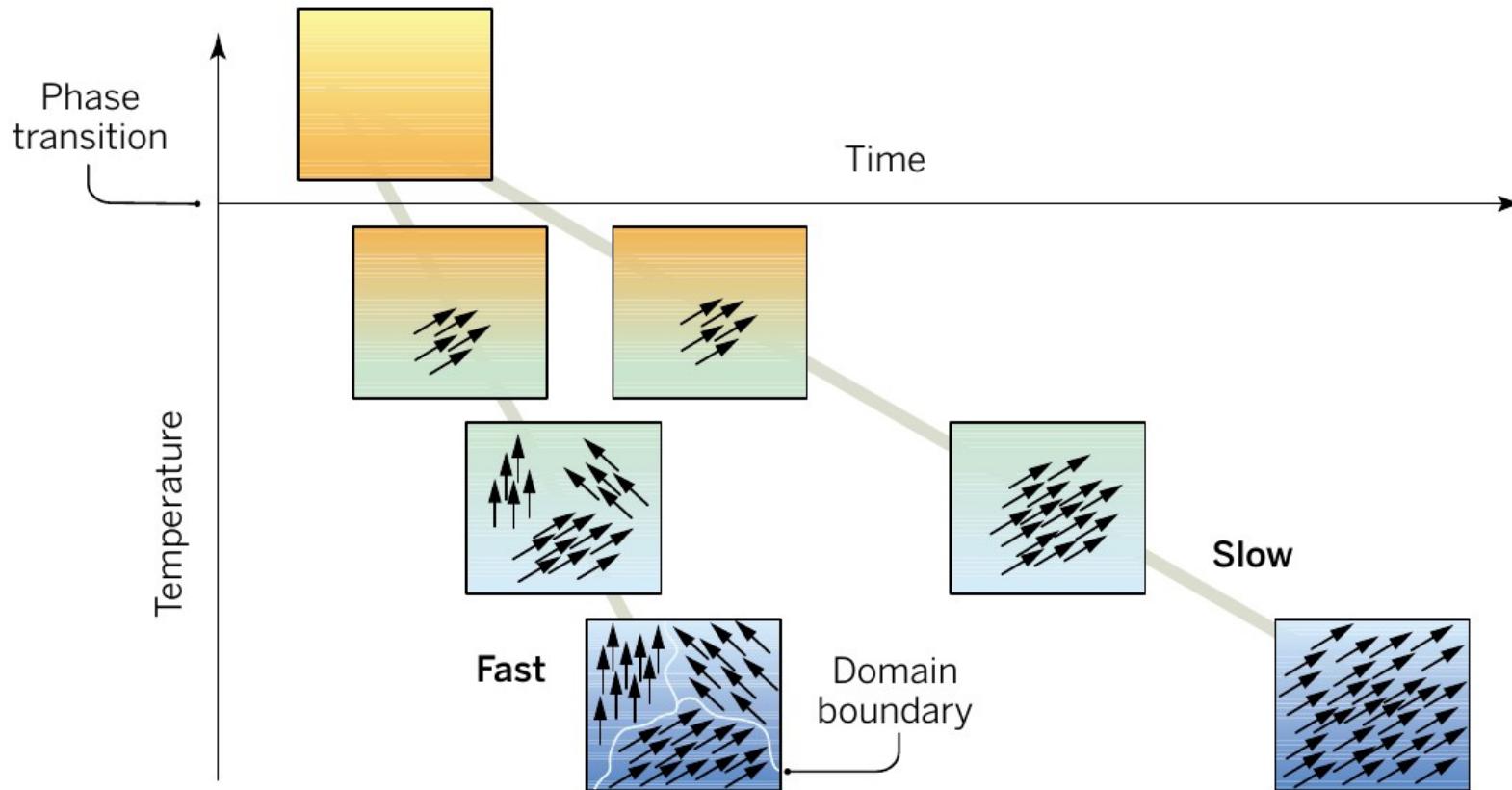
*Journées GdR Atomes Froids et IFRAF  
École Normale Supérieur, Paris  
5 November 2015*



# outline

- Introduction to the Kibble-Zurek mechanism
- Creating defects in Bose condensates via the Kibble-Zurek mechanism
- Defect's characterization
- Dynamics & interactions

# the Kibble-Zurek mechanism



- Second-order phase transitions
- Finite rate crossing
- Spontaneous and stochastic production of defects

# the Kibble-Zurek mechanism

Power-law scaling

coherence length

$$\xi(t) = \frac{\xi_0}{|\varepsilon(t)|^\nu}$$

reduced parameter:

$$\varepsilon = \frac{\lambda_c - \lambda}{\lambda_c}$$

Case of linear quench

$$\varepsilon(t) = t/\tau_Q$$

relaxation time

$$\tau(t) = \frac{\tau_0}{|\varepsilon(t)|^{z\nu}}$$

time to the transition

$$\tau(t) \approx |\varepsilon/\dot{\varepsilon}|$$

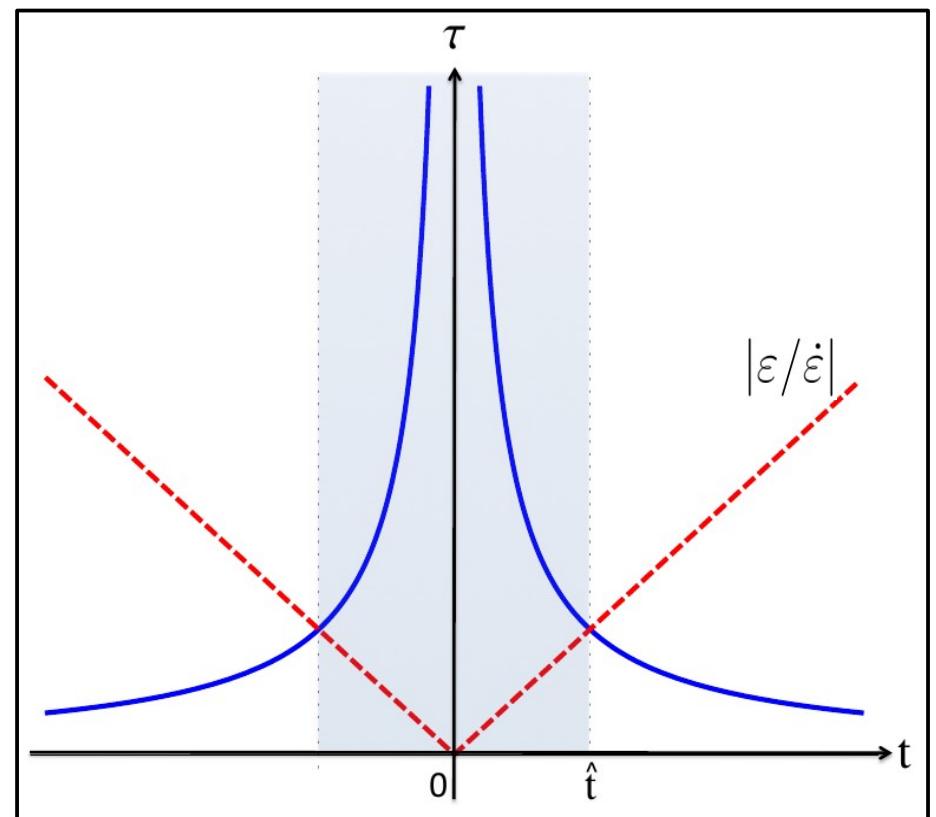
“freezing time”

$$\hat{t} \sim (\tau_0 \tau_Q^{z\nu})^{\frac{1}{1+z\nu}}$$

domain size

$$\hat{\xi} = \xi(\hat{t}) = \xi_0 \left( \frac{\tau_Q}{\tau_0} \right)^{\frac{\nu}{1+z\nu}}$$

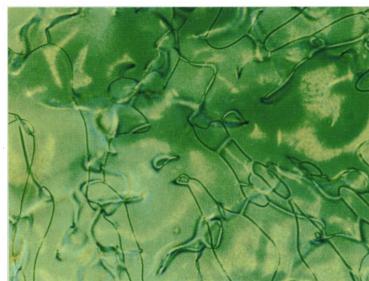
**ADIABATIC -- IMPULSE -- ADIABATIC**



density of defects

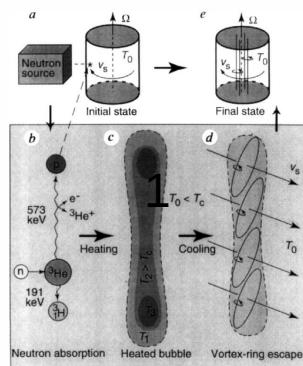
$$d \sim \hat{\xi}^{-D}$$

## Liquid crystals: isotropic/nematic



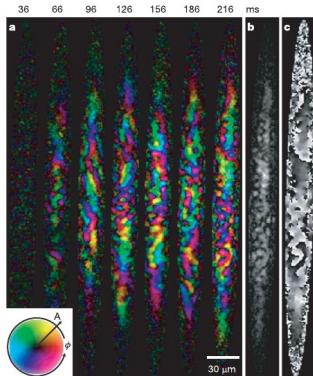
I. Chuang et al. (1991)

## Liquid $^3\text{He}$ : normal/SF



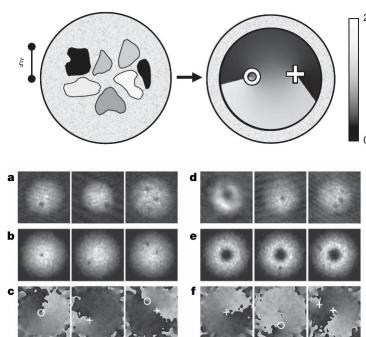
C. Bauerle et al. (1996)  
V.M.H Ruutu et al. (1996)

## Bose gases: ferromagnetic



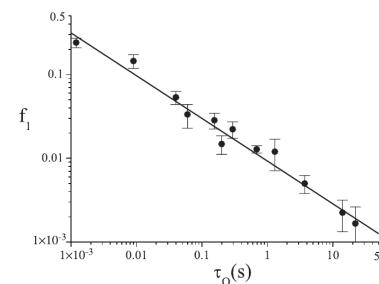
L. E. Sadler et al. (2006)

## Bose gases: thermal/BEC



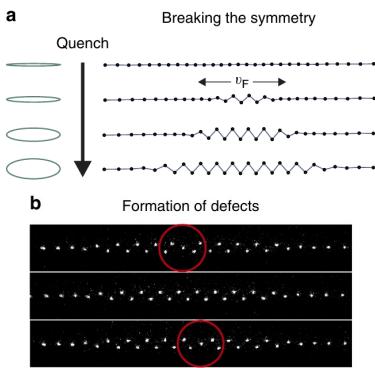
C.N. Weiler et al. (2008)

## annular Josephson junctions



R. Monaco et al. (2009)

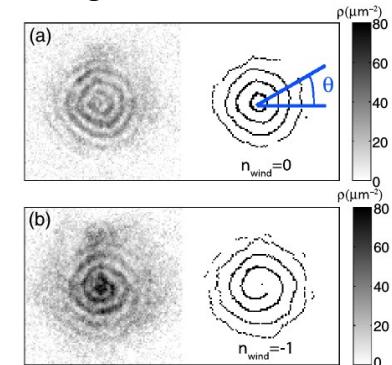
## 1D ion crystals: linear/zig-zag



S. Ulm et al. (2013)

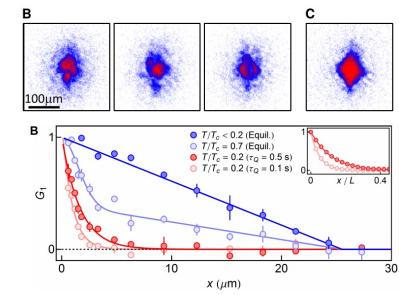
K. Pyka et al. (2013)

## Bose gases: thermal/BEC, $D < 3$



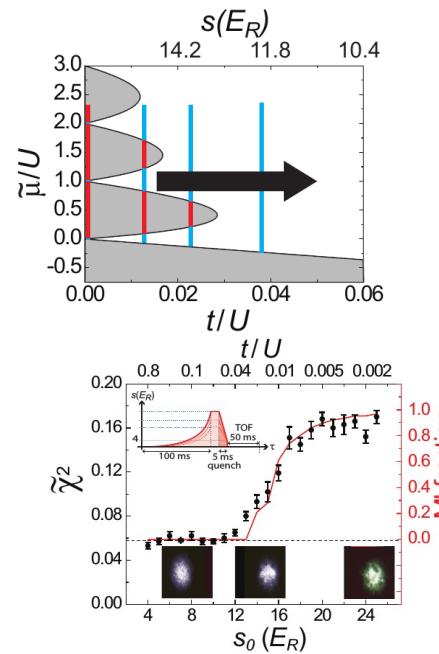
L. Corman et al. (2014)

## Hom. Bose gases: thermal/BEC



N. Navon et al. (2015)

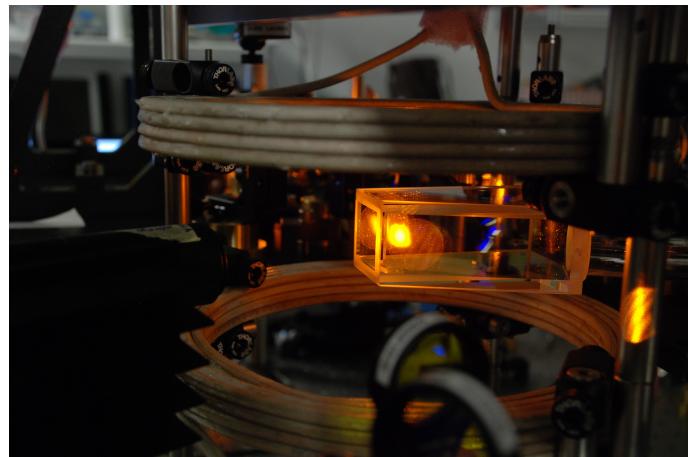
## T=0 Bose gases: Mott/SF



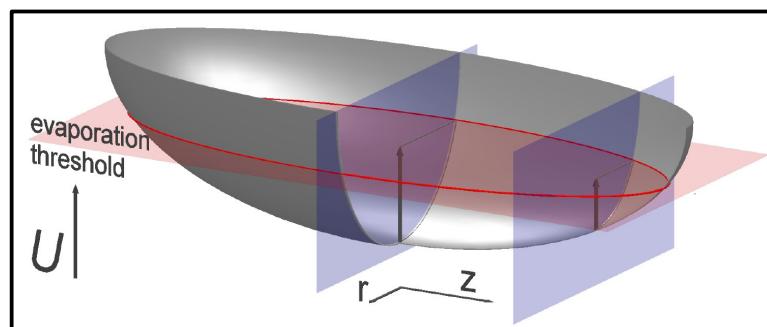
D. Chen et al. (2011)

S. Braun et al. (2014)

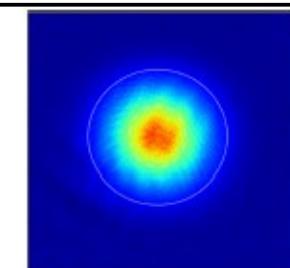
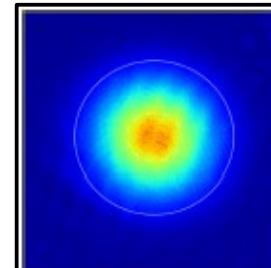
# experimental setup in Trento



G. Lamporesi *et al.*,  
Rev. Sci. Instrum. **84**, 063102 (2013)

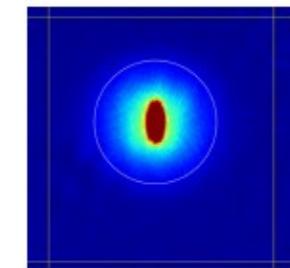
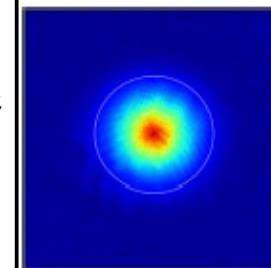


$T=1.1 \mu\text{K}$   
 $N=2.5 \cdot 10^7$



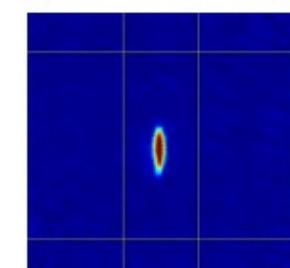
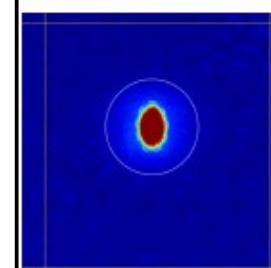
$T=870 \text{ nK}$   
 $N=2 \cdot 10^7$

$T=650 \text{ nK}$   
 $N=1.7 \cdot 10^7$



$T=470 \text{ nK}$   
 $N=1,1 \cdot 10^7$

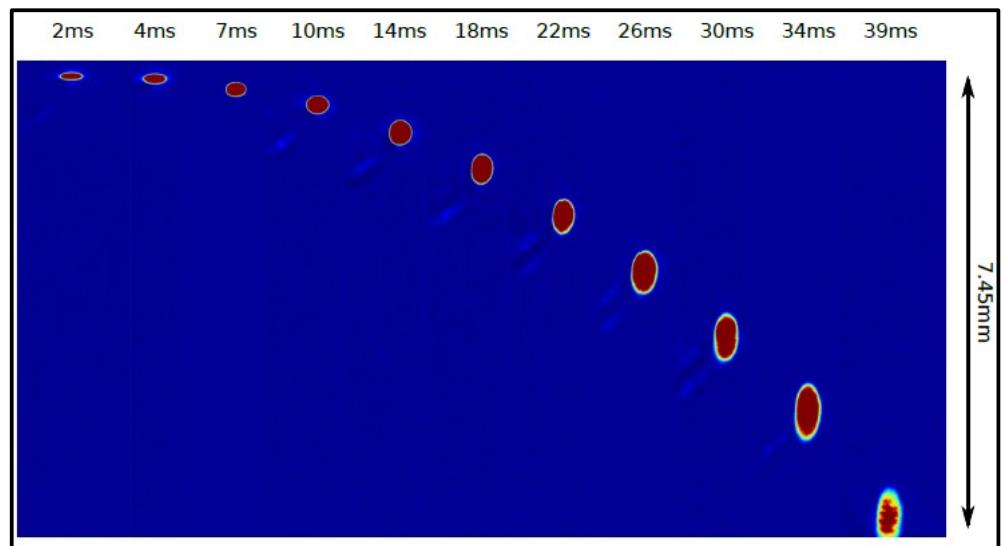
$T=290 \text{ nK}$   
 $N=7 \cdot 10^6$



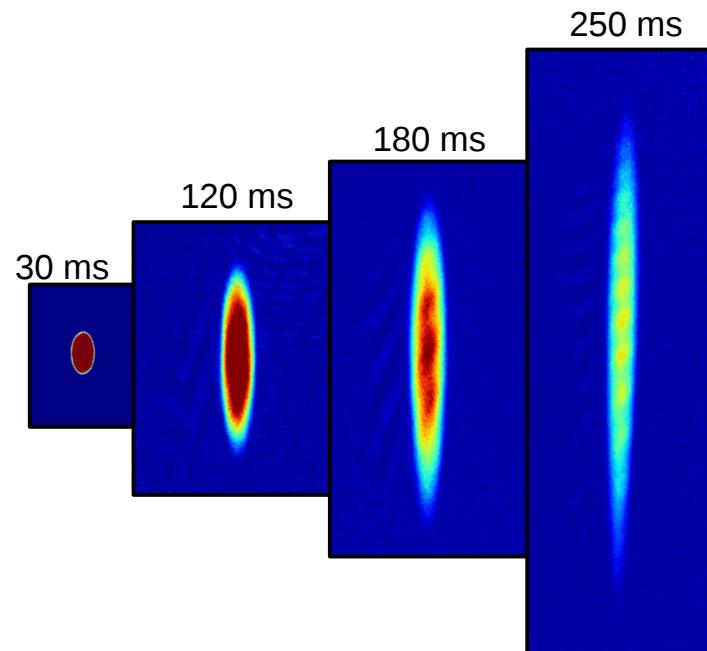
$T < 200 \text{ nK}$   
 $N=4 \cdot 10^6$

# ToF expansion of a BEC

expansion time limited to  $\sim 40$  ms  
due to the gravity fall

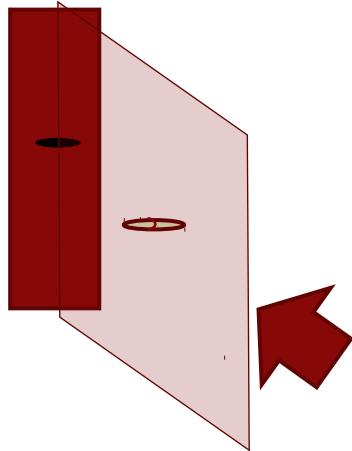


magnetic levitation against the gravity  
to increase the expansion time

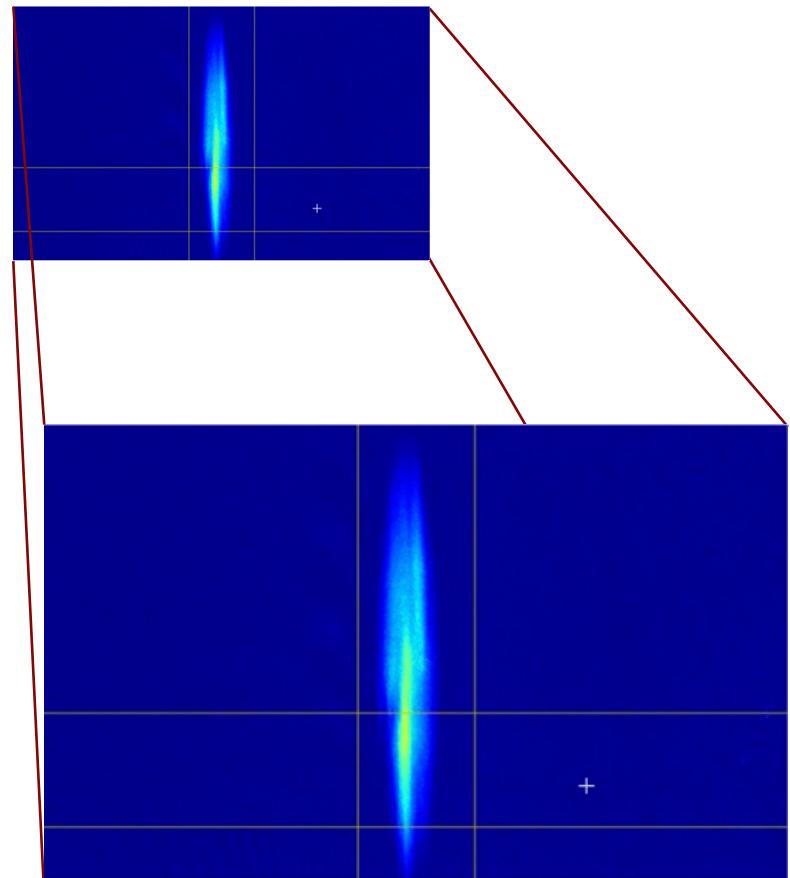
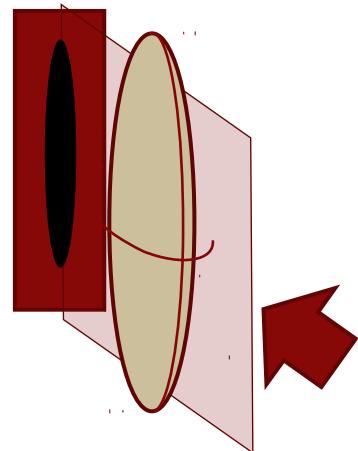


# ToF expansion of a BEC

In trap

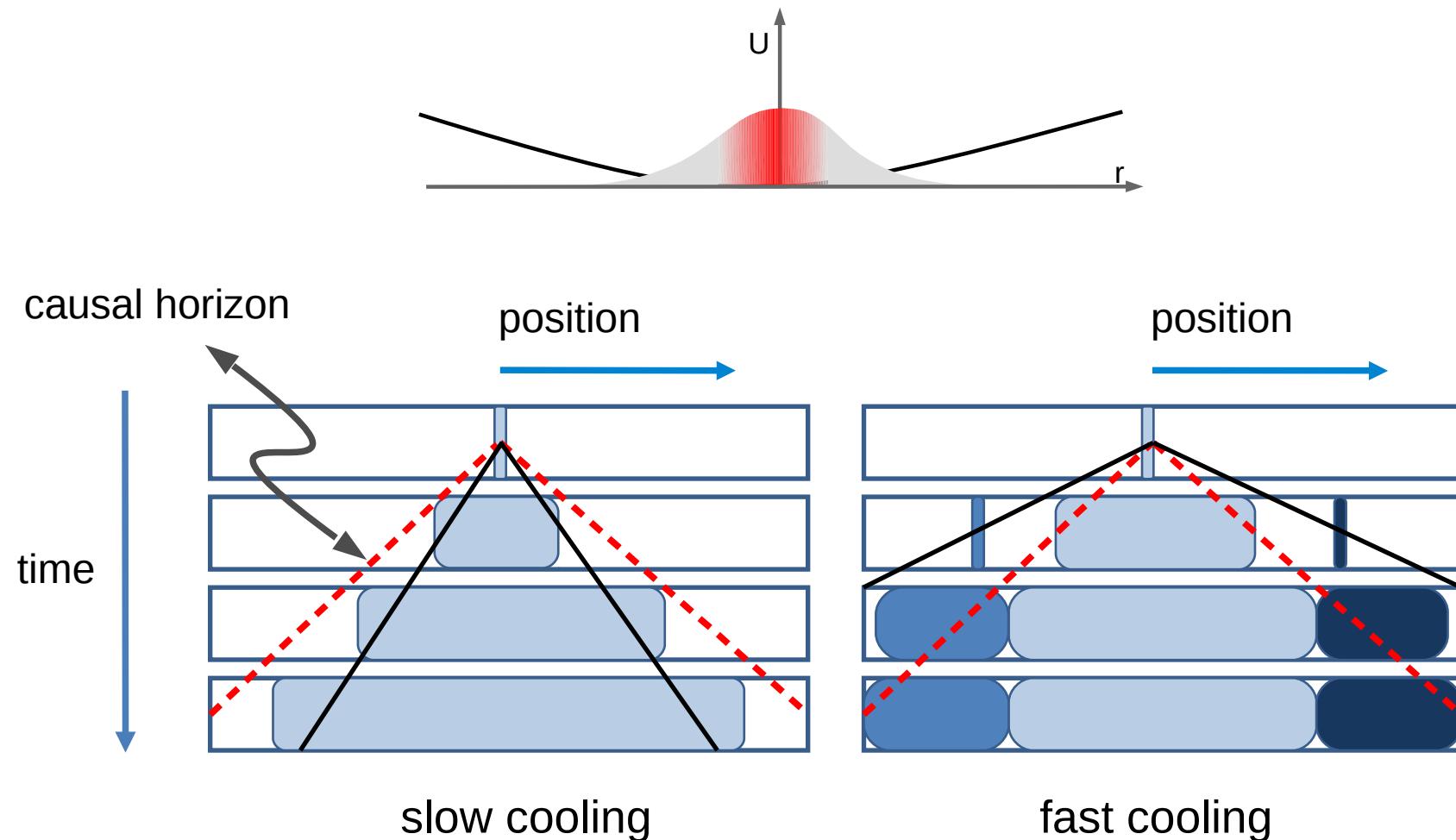


after expansion



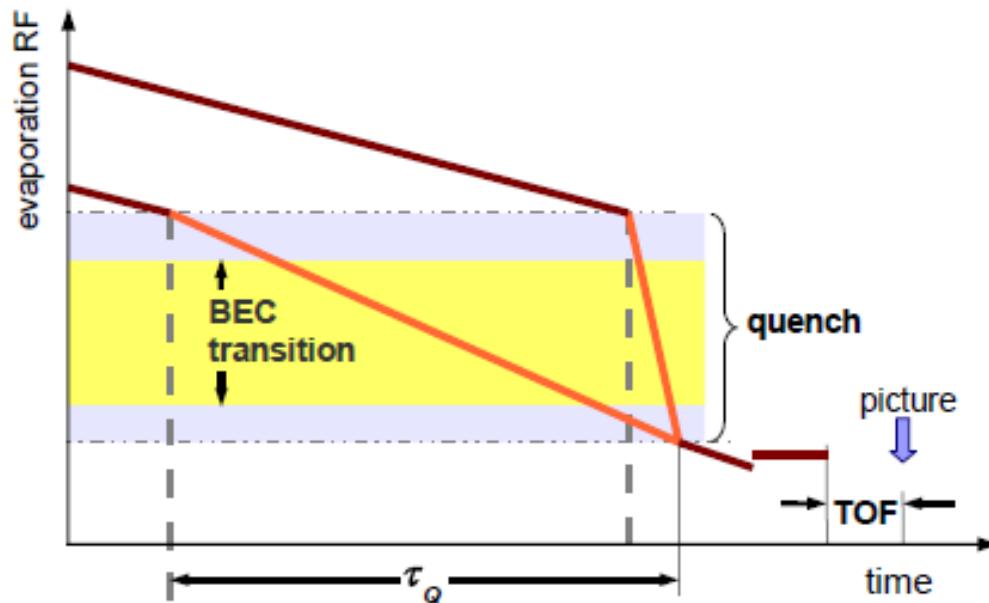
**Key observation:** the number of defects strongly depends on the rate at which the BEC transition is crossed !!

# Generating solitons via the Kibble-Zurek mechanism

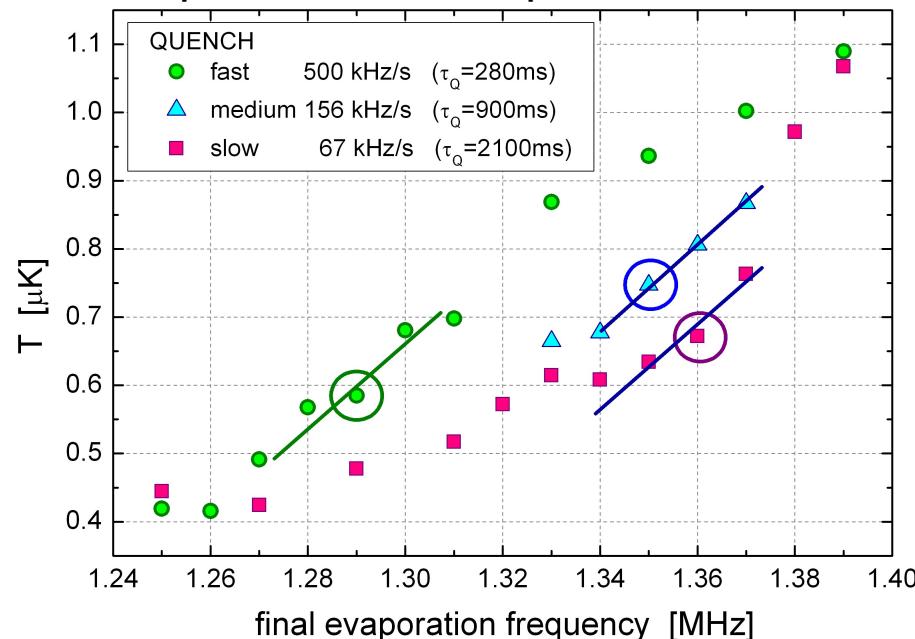


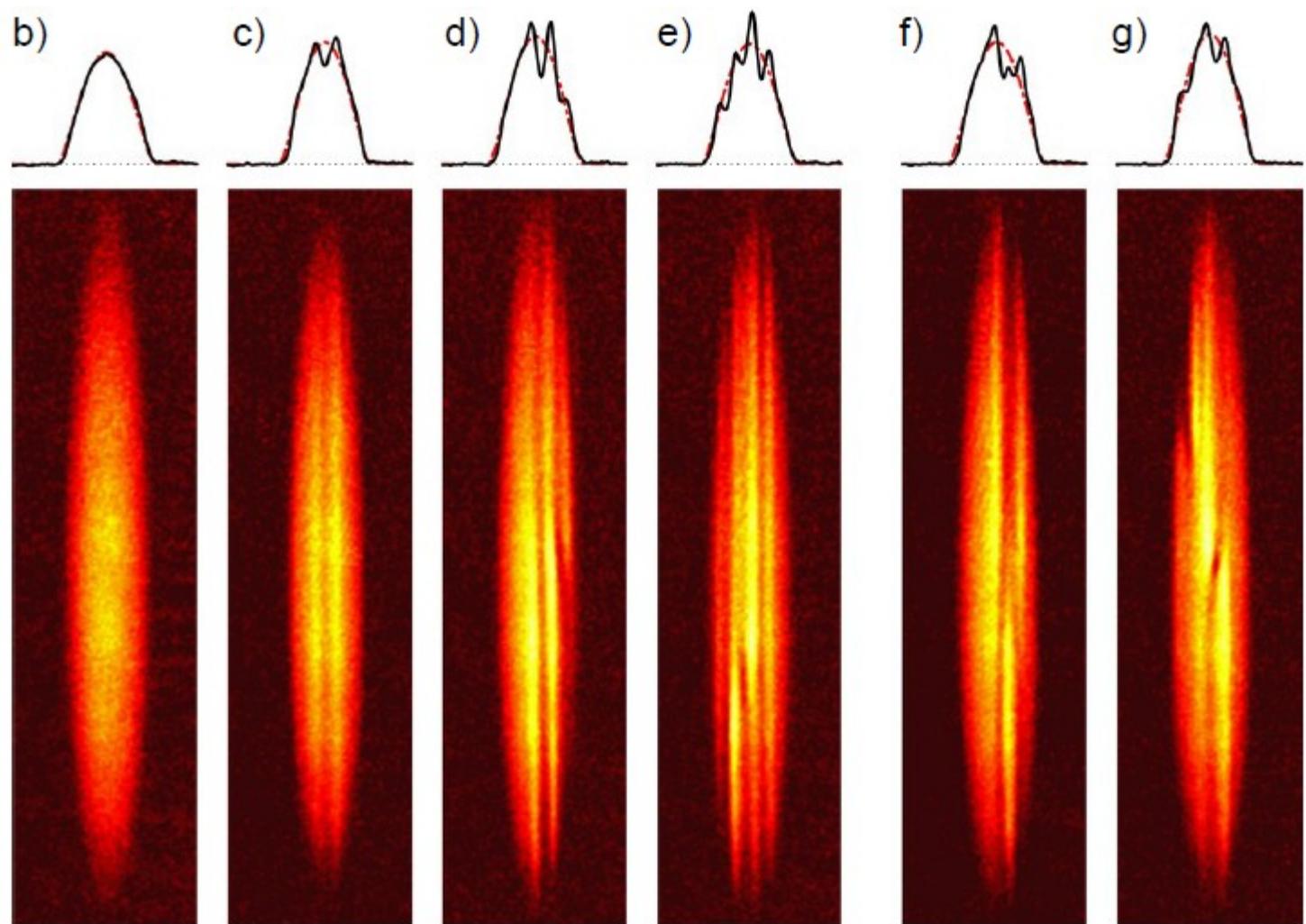
W. H. Zurek, PRL 102, 105702 (2009)

**Check:** change the cooling/quench time.

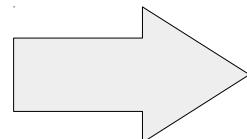


temperature VS evapoation thershold



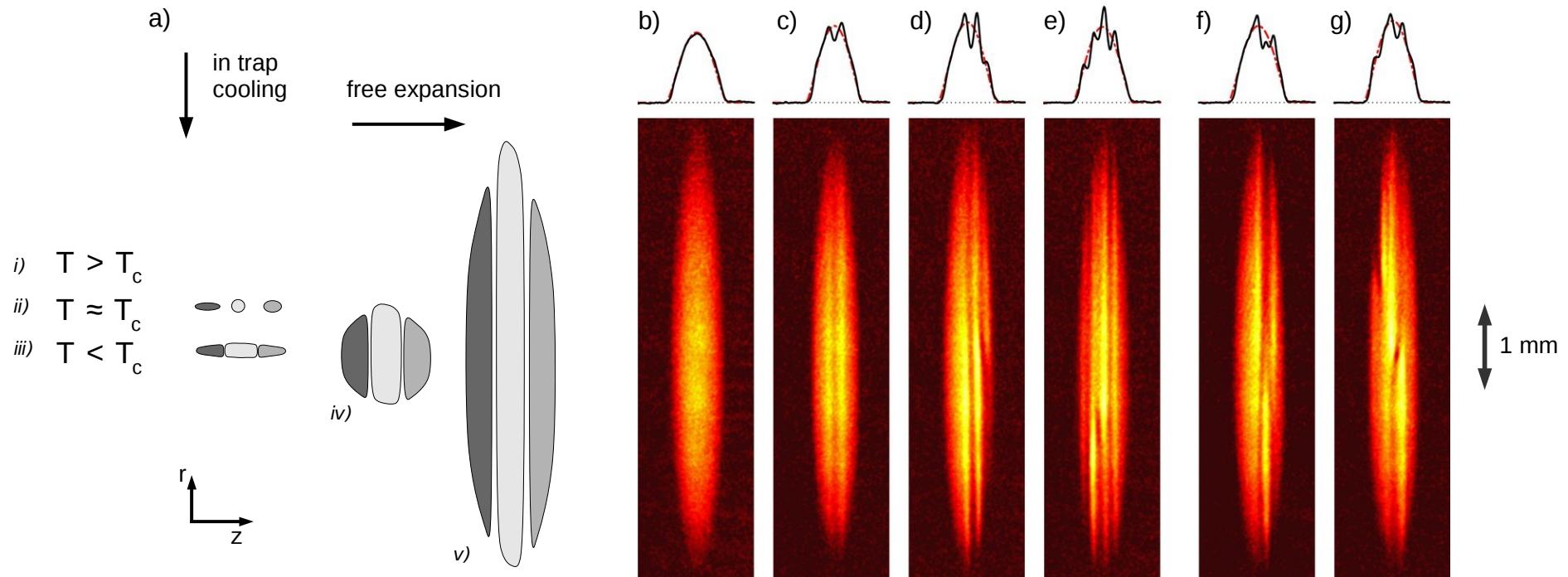


*slow cooling*

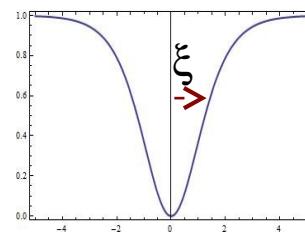


*fast cooling*

**guess:** these are gray solitons spontaneously nucleated at the BEC transition by the Kibble-Zurek mechanism (KZM) !!



imaging resolution:  $10 \mu\text{m}$   
 soliton width in trap:  $\xi(0) = 200\text{-}250 \text{ nm}$   
 width after TOF:  $\xi(180 \text{ ms}) = 50\text{-}100 \mu\text{m}$



# Measurement of the KZ $\alpha$ coefficient

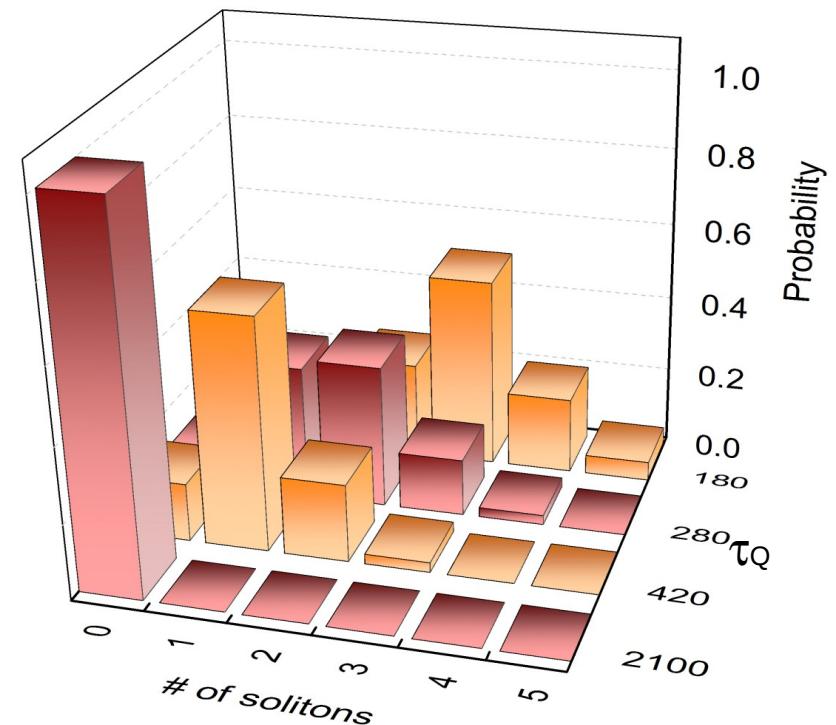
the number of defects is expected to follow a power-law as a function of the quench time (fixed size of the system)

$$N_s \mu \tau_Q^{-\alpha}$$

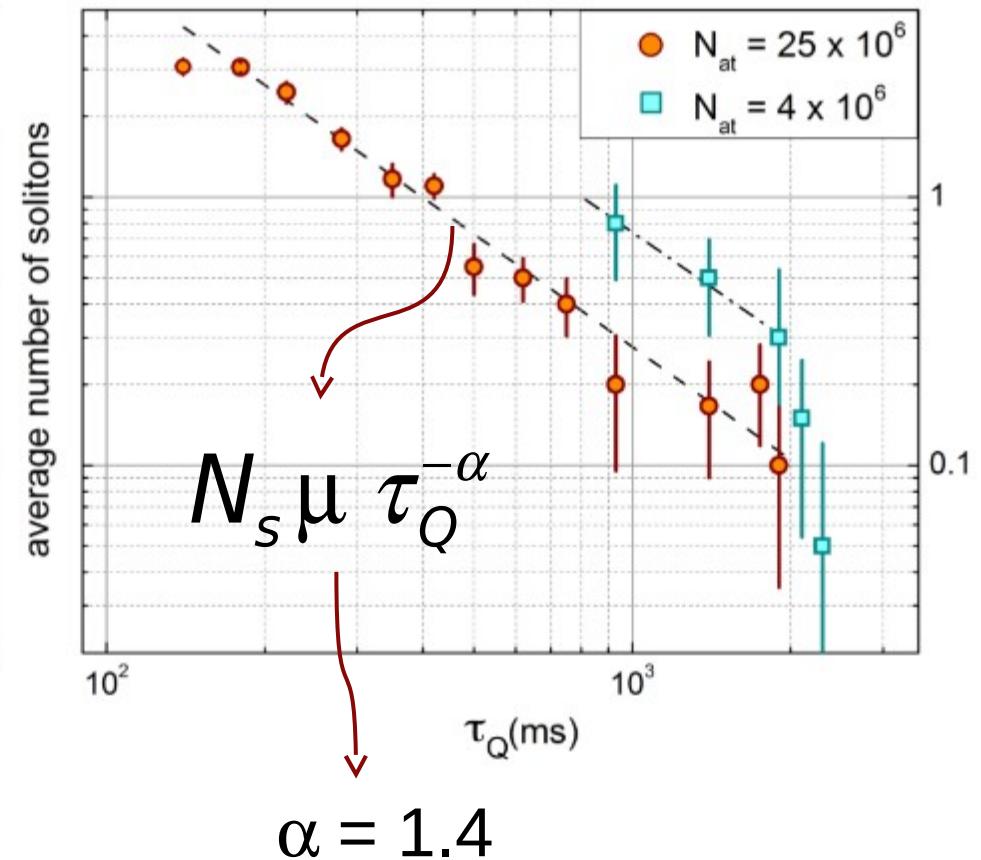
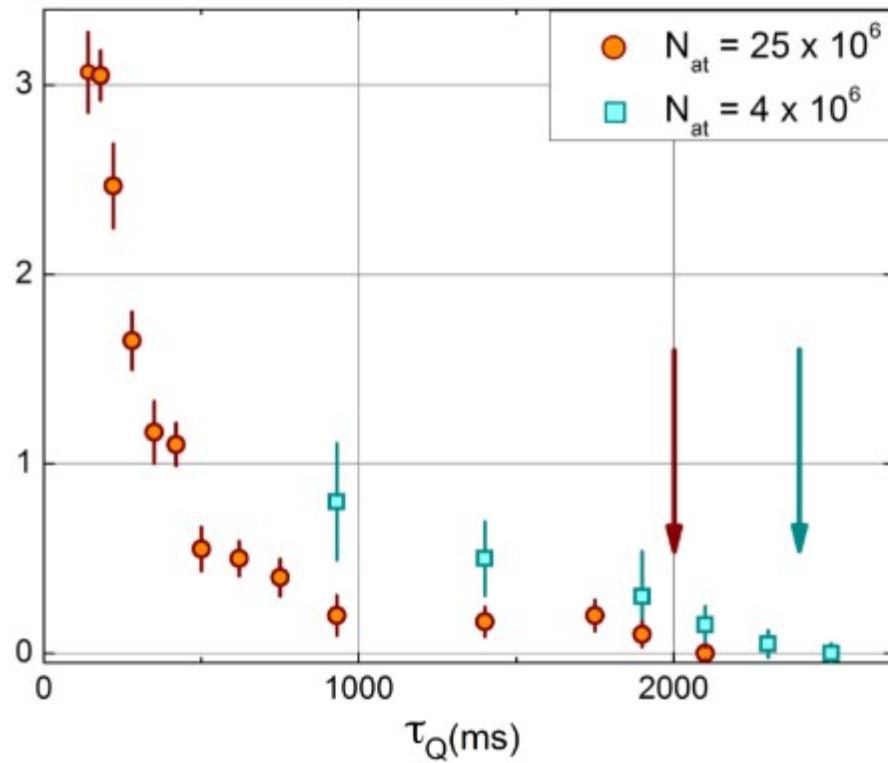
where  $\alpha$  is determined by the critical exponents of the phase transition.

W. H. Zurek  
PRL 102, 105702 (2009)

**OK**, we can count our solitons !



# Measurement of the KZ $\alpha$ coefficient

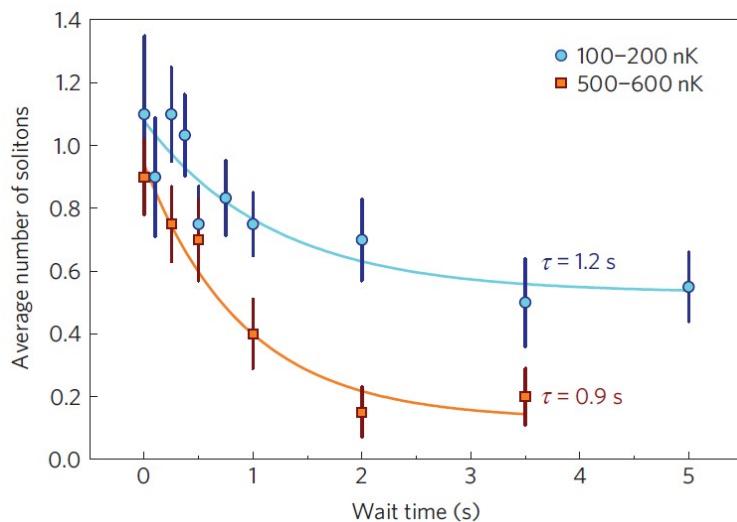


to compare with the available theoretical prediction (Zurek 2009)

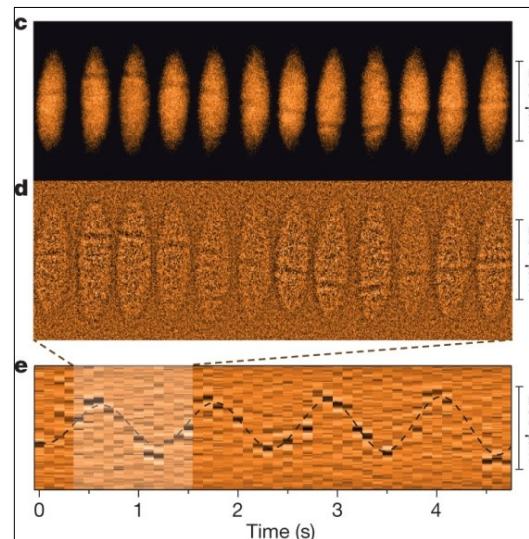
1D, homogeneous temperature

$$\alpha = 7/6 \sim 1.17$$

# The lifetime puzzle



(also in DFG at MIT)

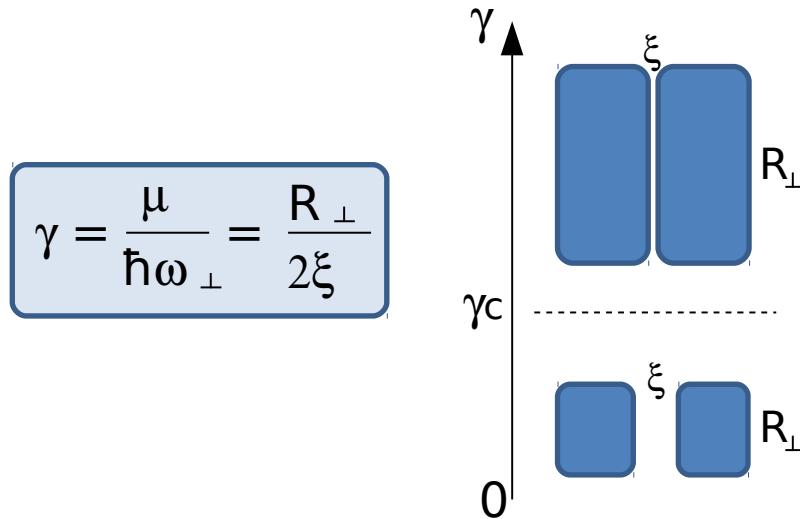


Yefsah et al.,  
Nature **499**, 426 (2013)

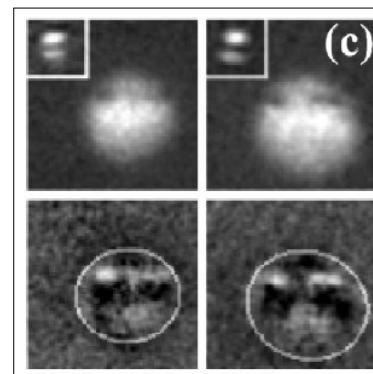
**Solitons** are expected to be **unstable**

THERMALLY (unless at T=0)

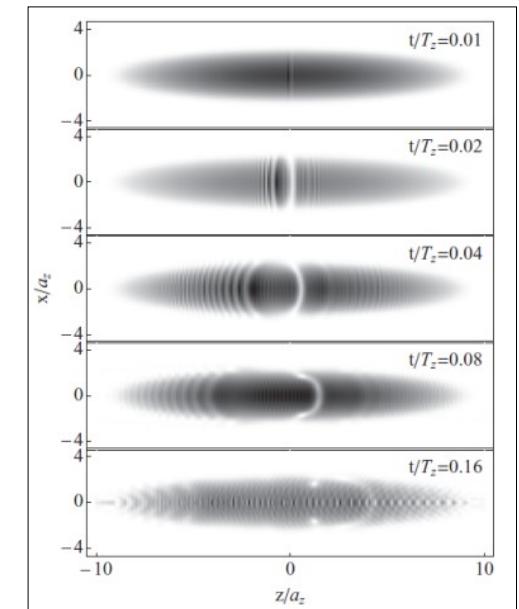
DYNAMICALLY (due to snake instabilities)



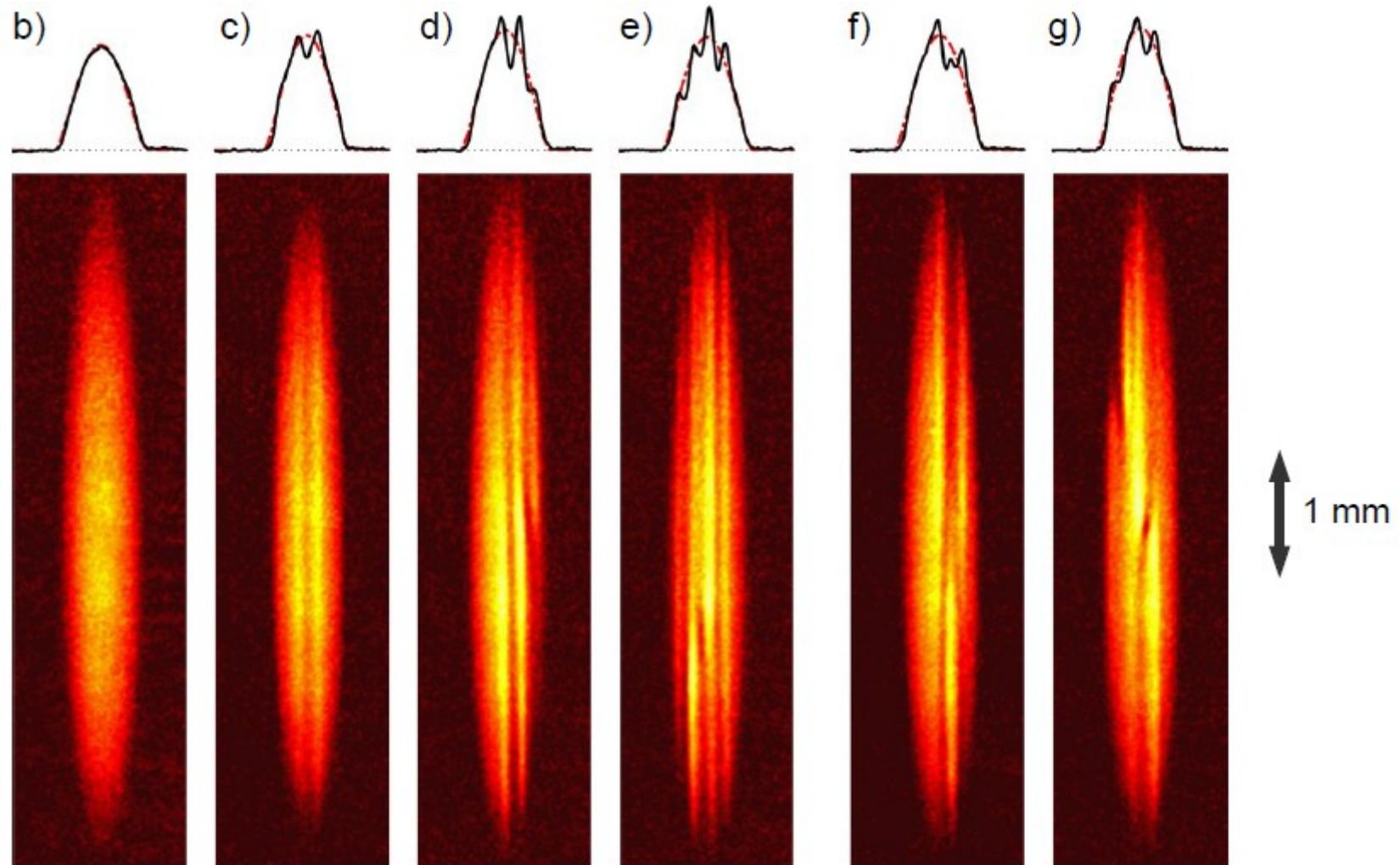
... and to decay into  
**vortex rings**

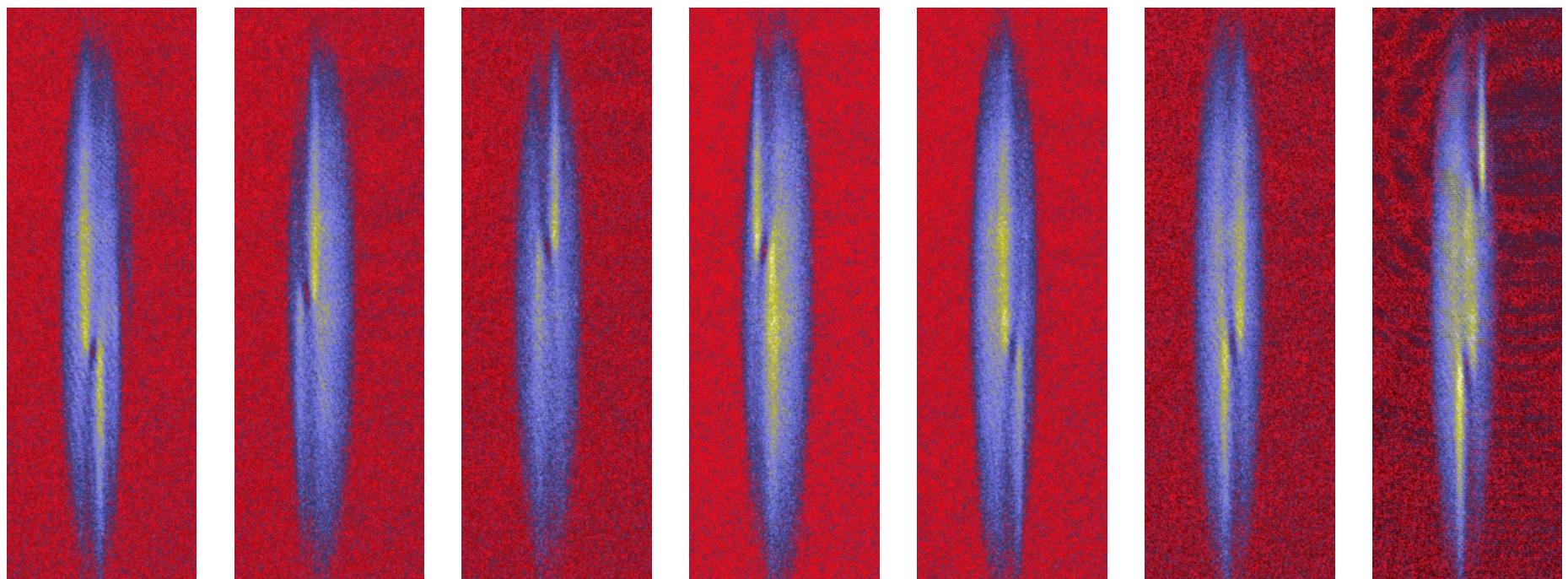
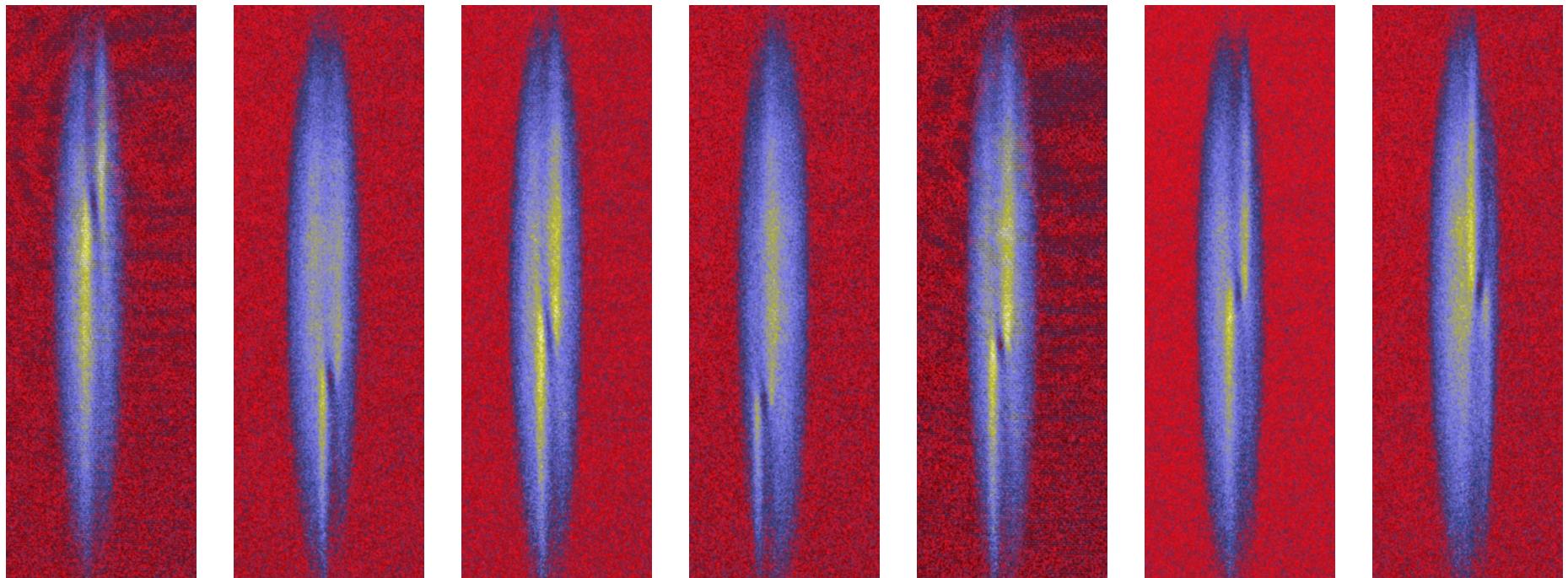


spherical BEC (JILA)  
Anderson et al.,  
PRL **86** 2926 (2001)



Reichl et al.,  
PRA **88**, 053626 (2013)

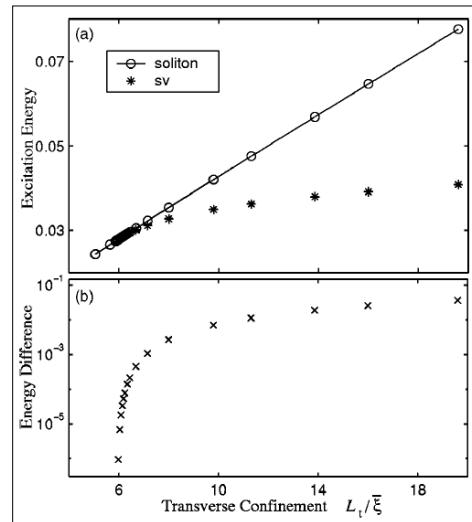
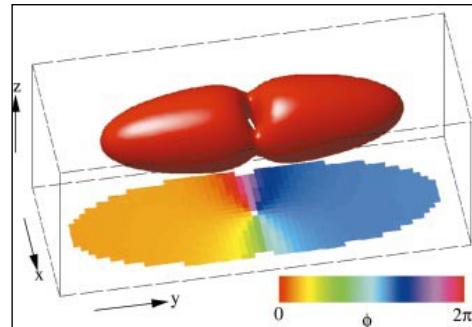
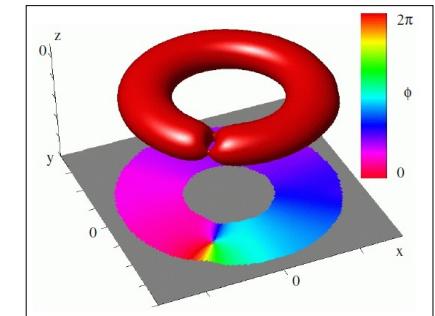




# Solitonic vortices

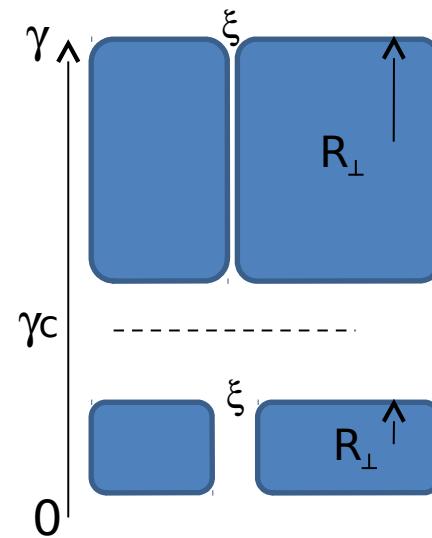
Vortex oriented perpendicularly to the axis of an axisymmetric elongated trap.

- Quantized vorticity
- Anisotropic phase pattern
- Planar density depletion

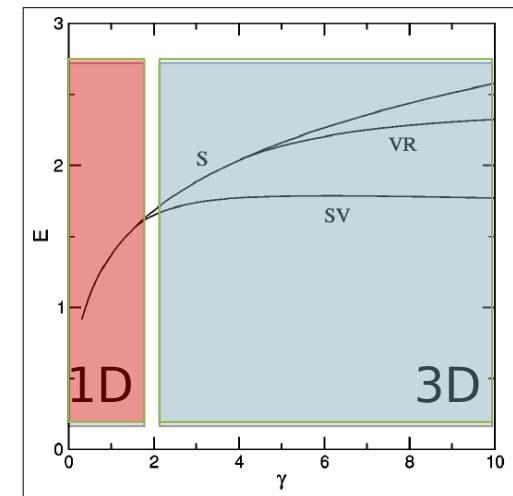


Brand et al., PRA **65**, 043612 (2002)

$$\gamma = \frac{\mu}{\hbar\omega_{\perp}} = \frac{R_{\perp}}{2\xi}$$



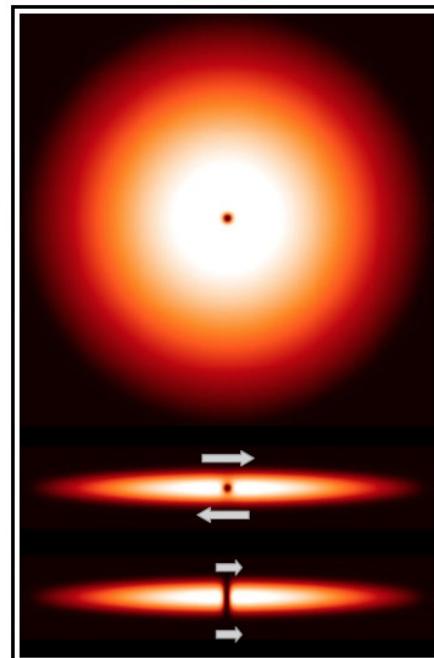
Brand et al., JPB **34**, L113 (2001)



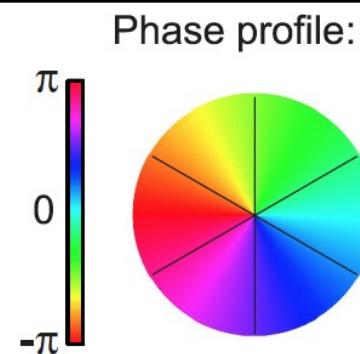
Komineas et al., PRA **68**, 043617 (2003)

# Solitonic vortices

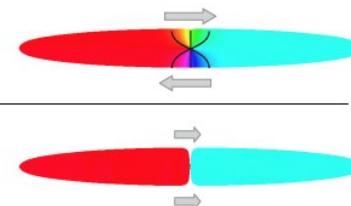
**Density in trap**



VORTEX



(b) SOLITONIC VORTEX  
in a cigar-shaped trap

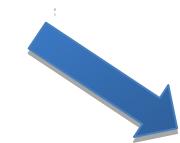
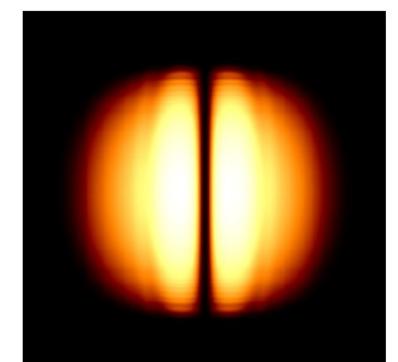
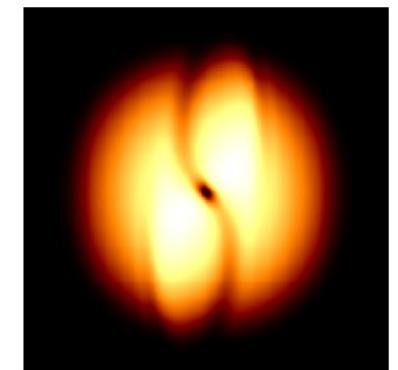


(c) SOLITON  
in a cigar-shaped trap

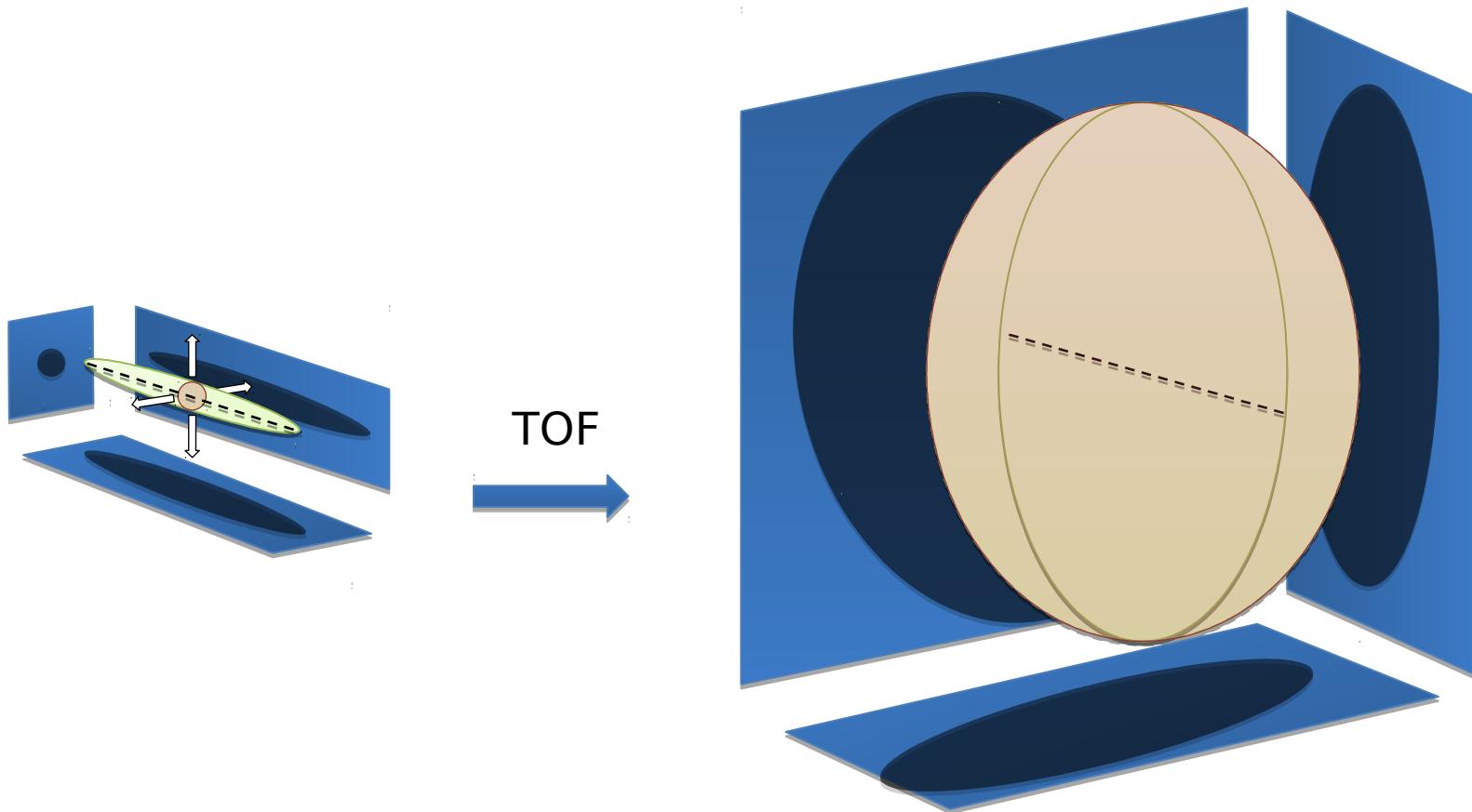
**Phase**

**Density  
after free  
expansion**

*Asymmetric  
twist*

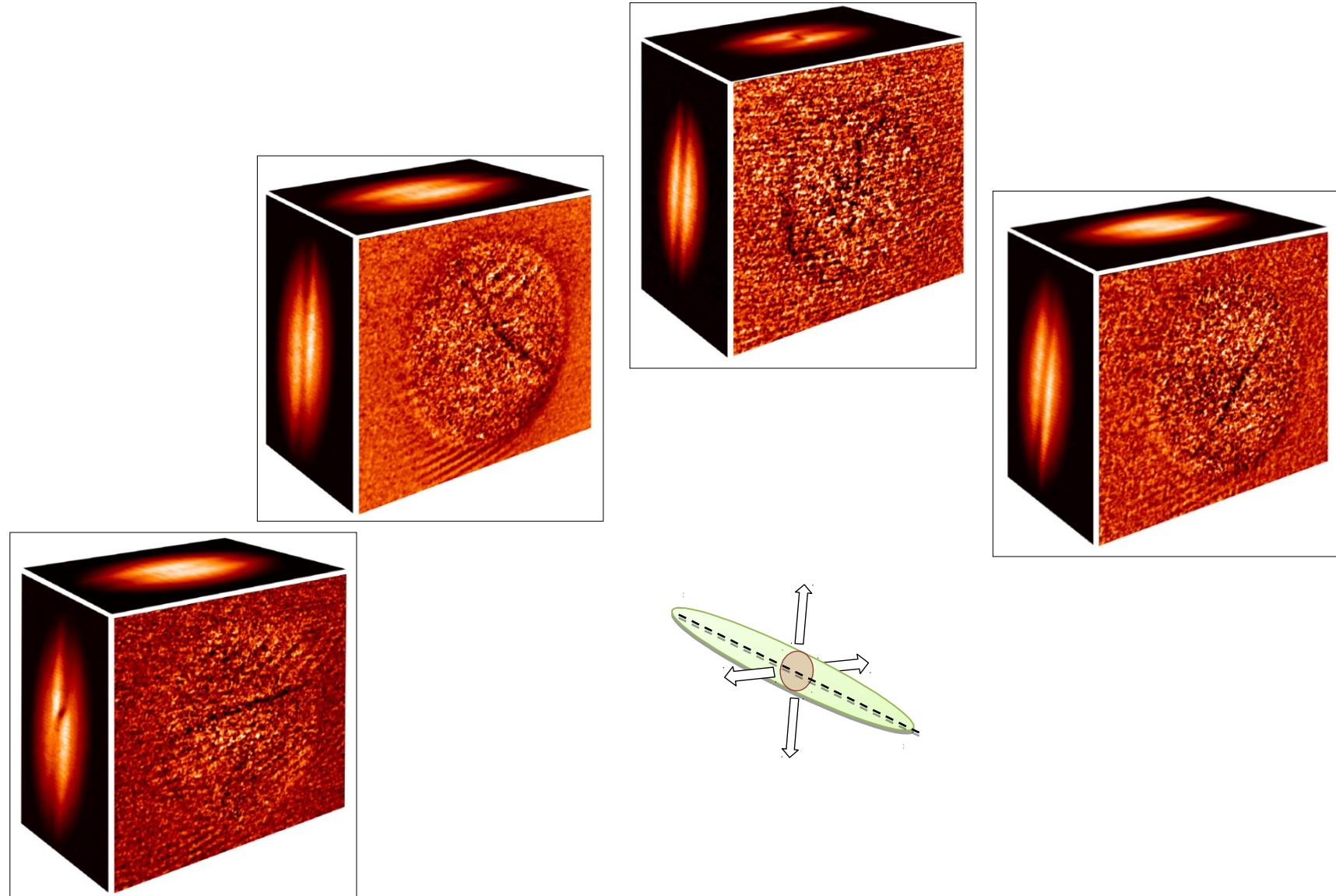


# Solitonic vortices



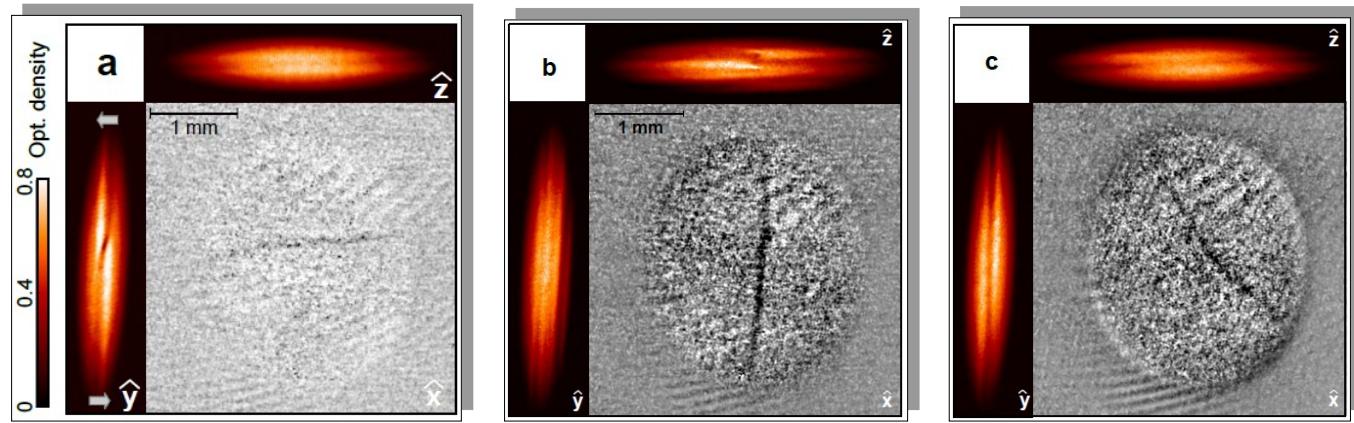
# Random orientation

Triaxial absorption imaging after long TOF

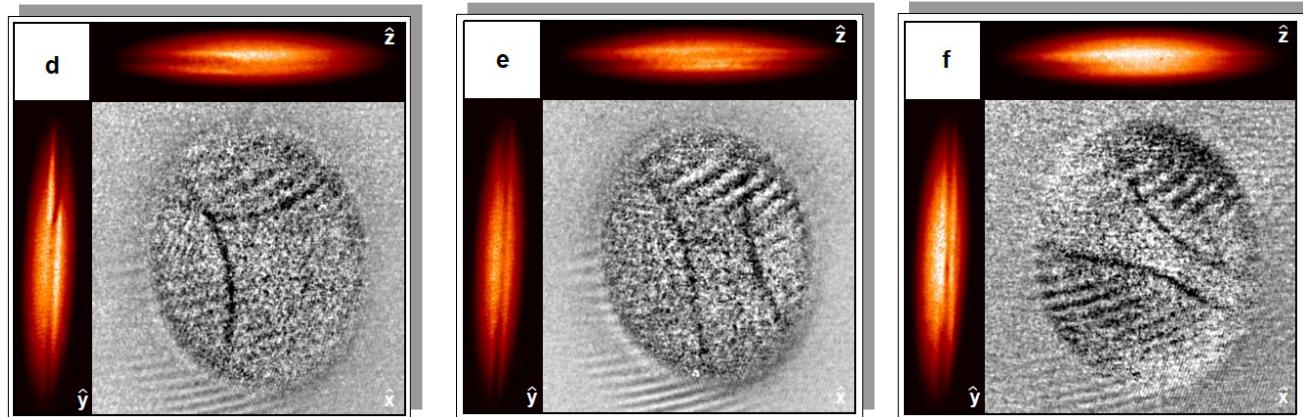


# Random number

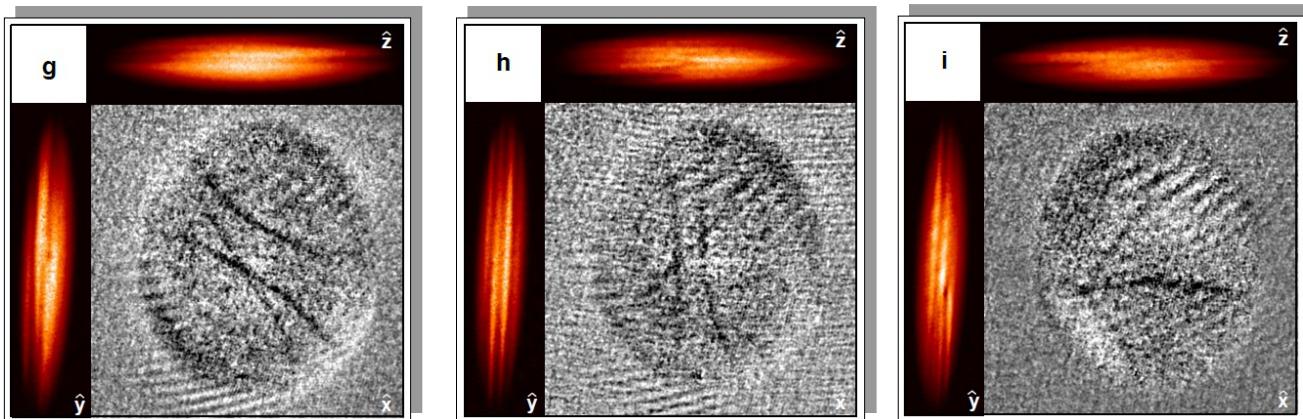
1



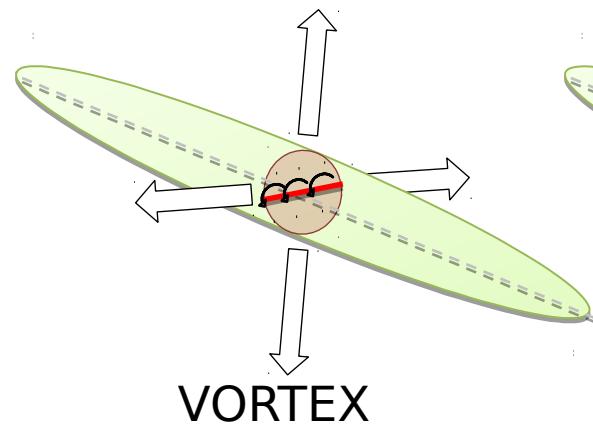
2



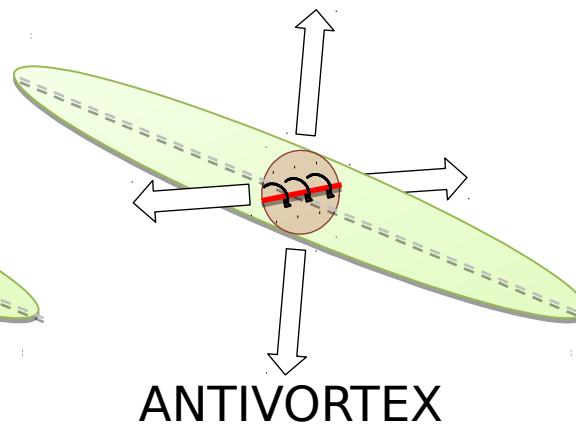
3



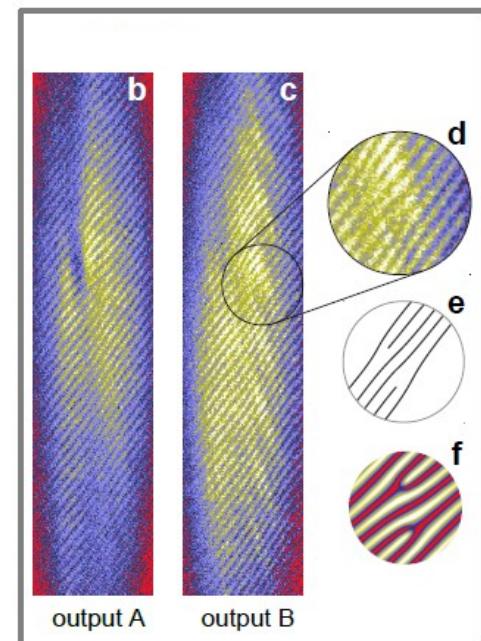
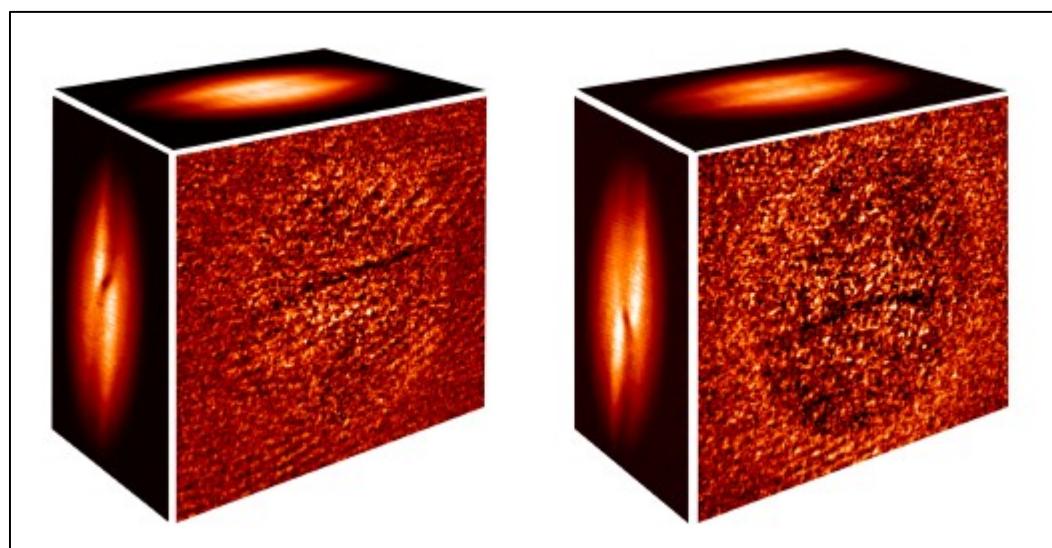
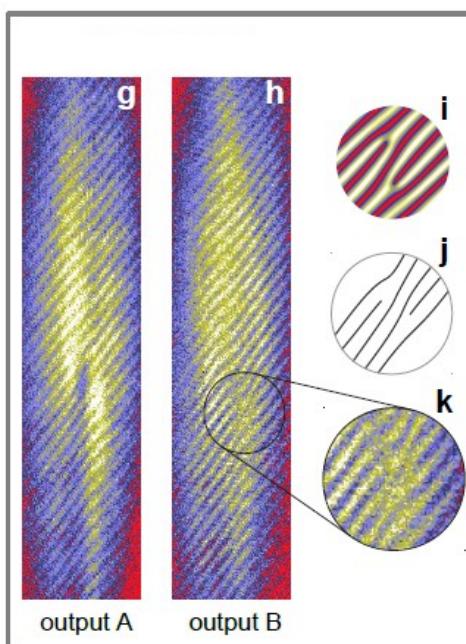
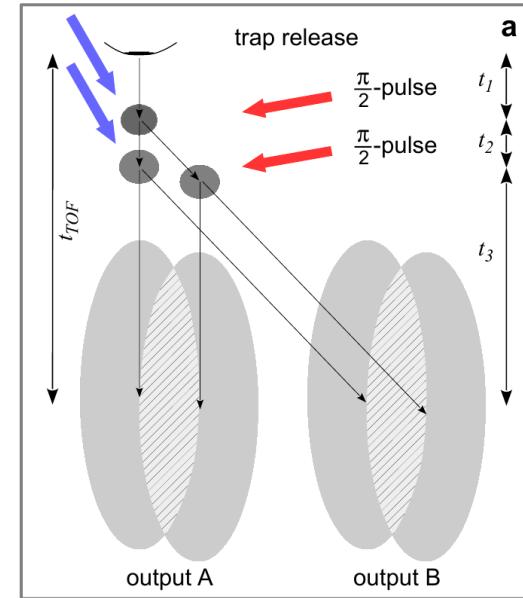
# Random circulation



VORTEX



ANTIVORTEX



homodyne detection of the phase pattern by interfering two copies of the condensate

S. Donadello *et al.*, PRL **113**, 065302 (2014)

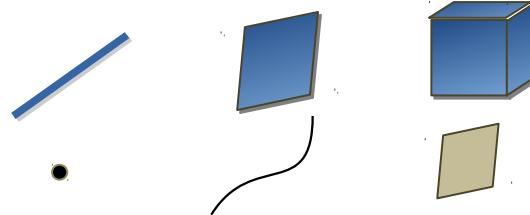
# Scaling exponents

$$n \sim \frac{\hat{\xi}^d}{\hat{\xi}^D} = \frac{1}{\xi_0^{D-d}} \left( \frac{\tau_0}{\tau_Q} \right)^{(D-d)\frac{\nu}{1+z\nu}}$$

$\nu, z$ : critical exponents

D: system dimension

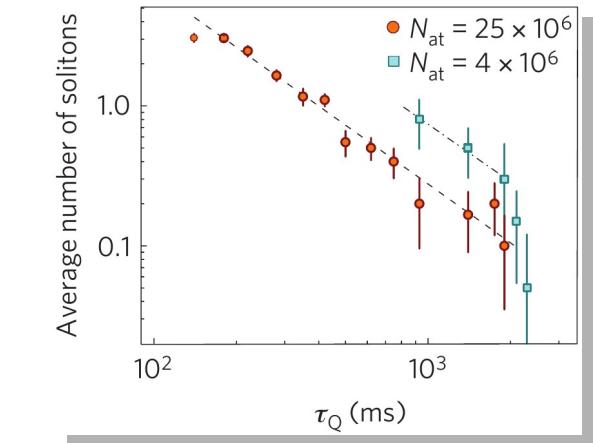
d: defect dimension



**(D-d)=2**

Critical exponents \ Trap	Homogeneous	Harmonic	Toroidal	Homog.	Harm.
Arbitrary ( $\nu, z$ )	$\frac{2\nu}{1+\nu z}$	$\frac{2(1+2\nu)}{1+\nu z}$	$\frac{1+3\nu}{1+\nu z}$	$\frac{\nu}{1+\nu z}$	$\frac{1+2\nu}{1+\nu z}$
Mean-field theory ( $\nu = \frac{1}{2}, z = 2$ )	$\frac{1}{2}$	2	$\frac{5}{4}$	1/4	1
Experiments/F model ( $\nu = \frac{2}{3}, z = \frac{3}{2}$ )	$\frac{2}{3}$	$\frac{7}{3}$	$\frac{3}{2}$	1/3	7/6

Del Campo *et al.*, NJP **13**, 083022 (2011)

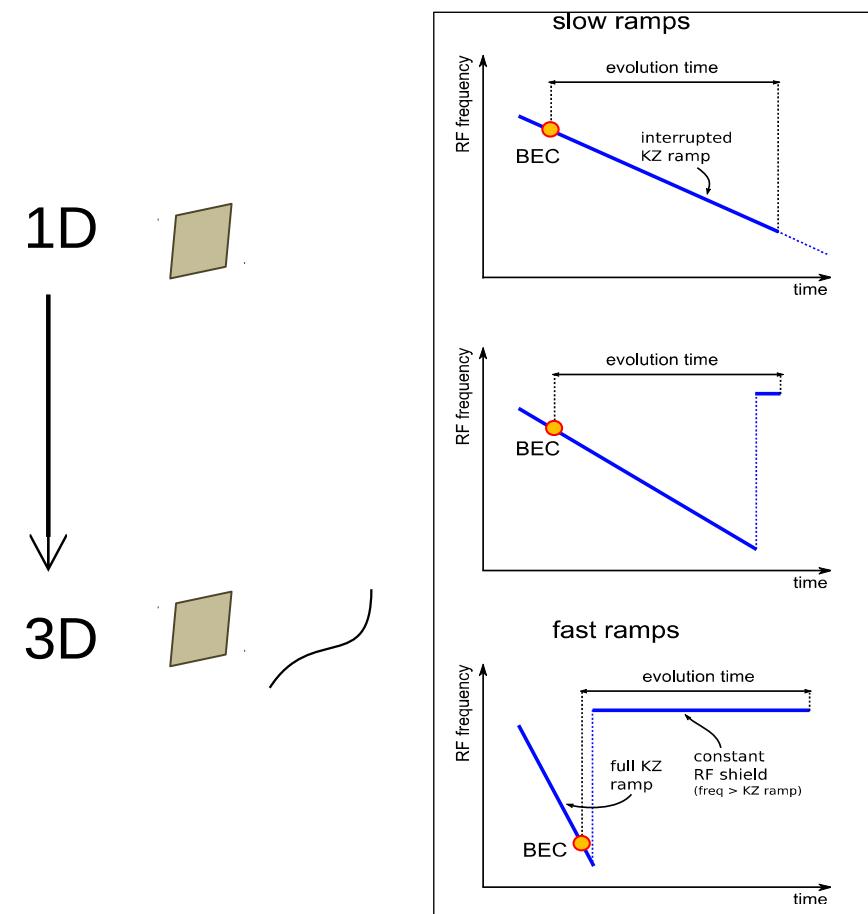
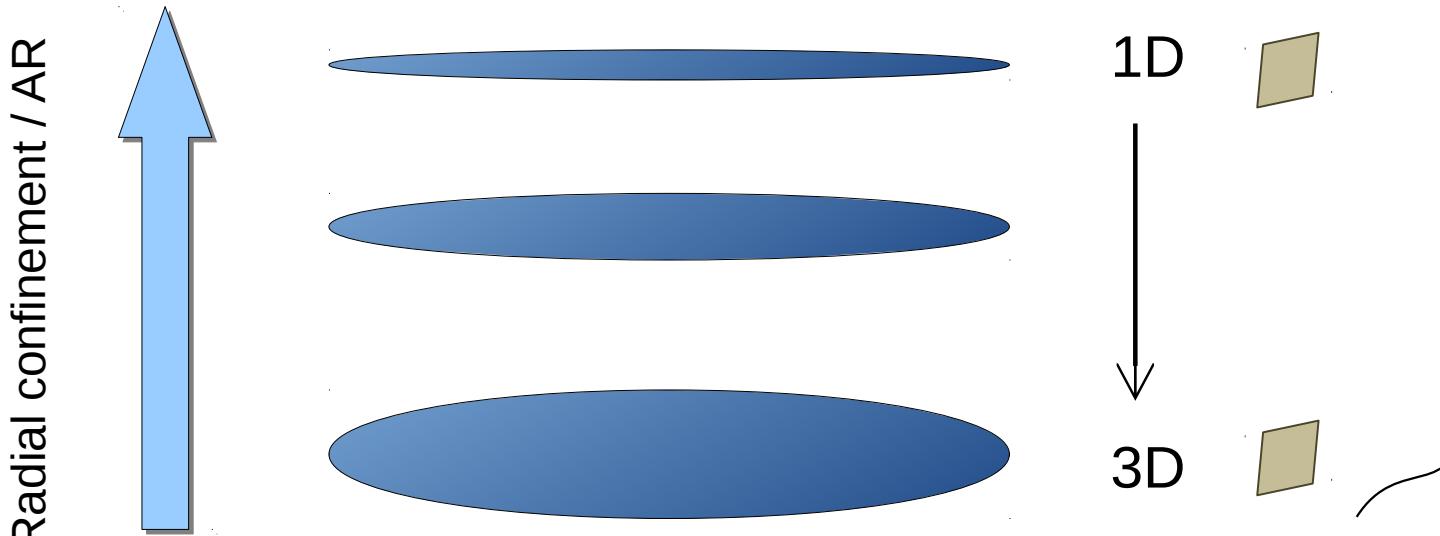


(  $\alpha=1.4$  )

Zurek, PRL **102**, 105702 (2009)

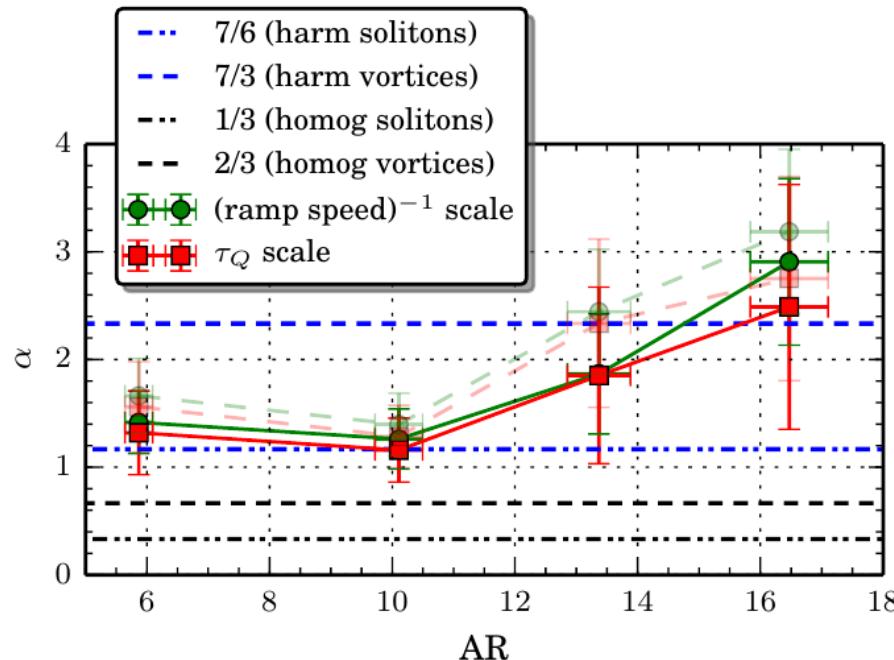
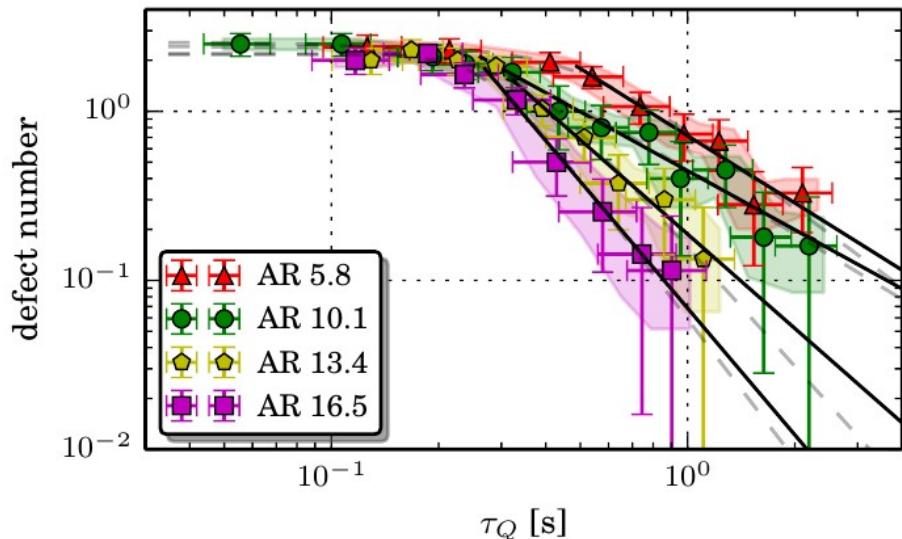
Search for effects on KZ scaling due to geometry of confinement:

- dimensional cross-over
- creation of different types of defects



Revised evaporation ramps suppressing the effects of decay of defects

# preliminary



- power-law scaling for slow ramps
- aspect ratio dependent exponent

- flat plateau for fast ramps
- plateau independent on aspect ratio

	model	$\alpha_{\text{homog}}$	$\alpha_{\text{harm}}$
solitons in 3D or in 1D $(S - D = 1)$	MF	$1/4$	$1$
	F-model	$1/3$	$7/6$
vortices in 3D $(S - D = 2)$	MF	$1/2$	$2$
	F-model	$2/3$	$7/3$

# Dynamics of quantized vortices

Determine dissipative and transport processes in:

- Superfluid helium
- Superconductors
- Neutron stars

*In atomic BECs:*

Controllable environment, spatial scale from  $\xi$  to tens of  $\xi$ ,  
inhomogeneous systems, boundary physics...

**BUT**

Vortices are produced stochastically and their dynamics cannot be followed through standard destructive absorption imaging

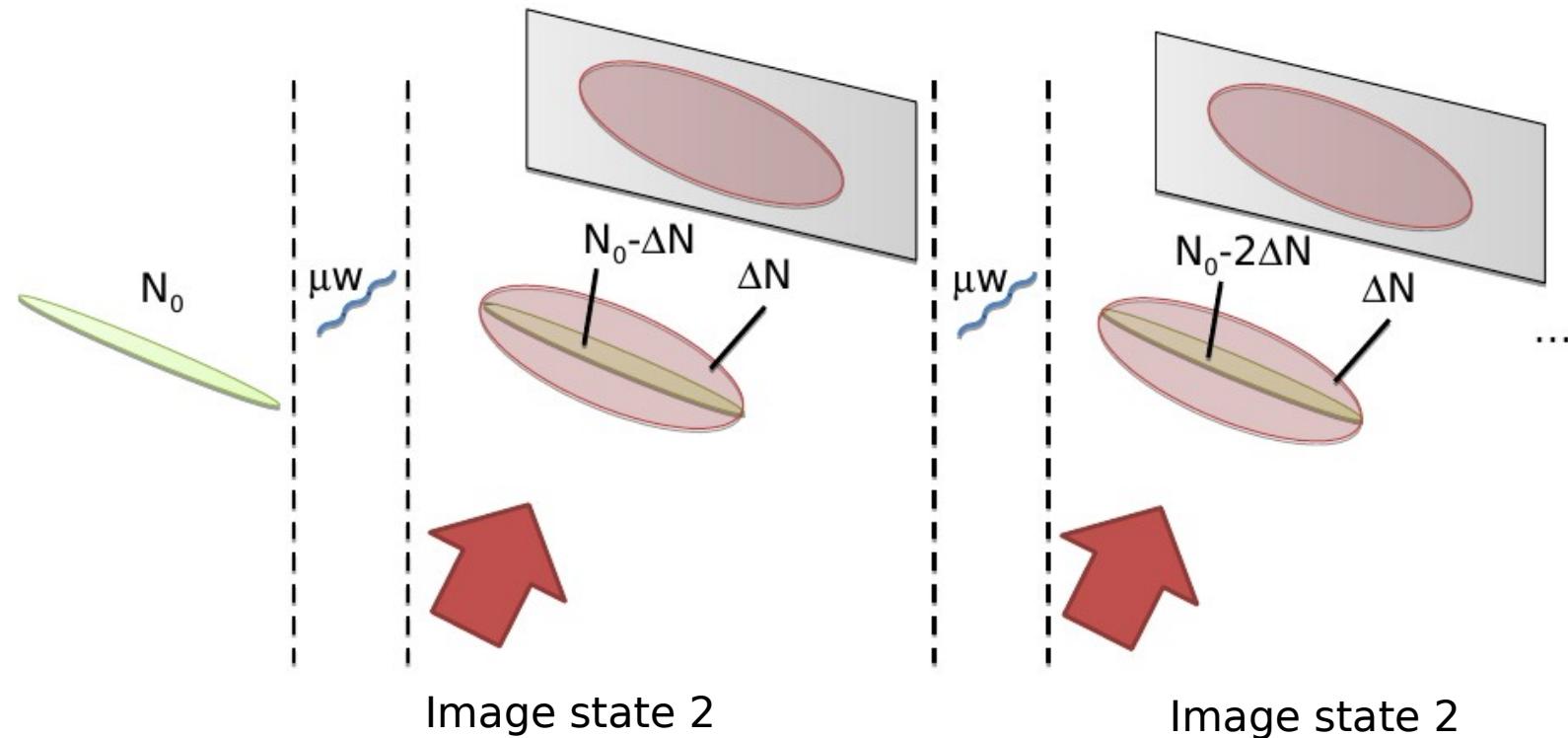
# Stroboscopic imaging of defect dynamics

$\mu$ -wave pulses extract a small fraction from the BEC

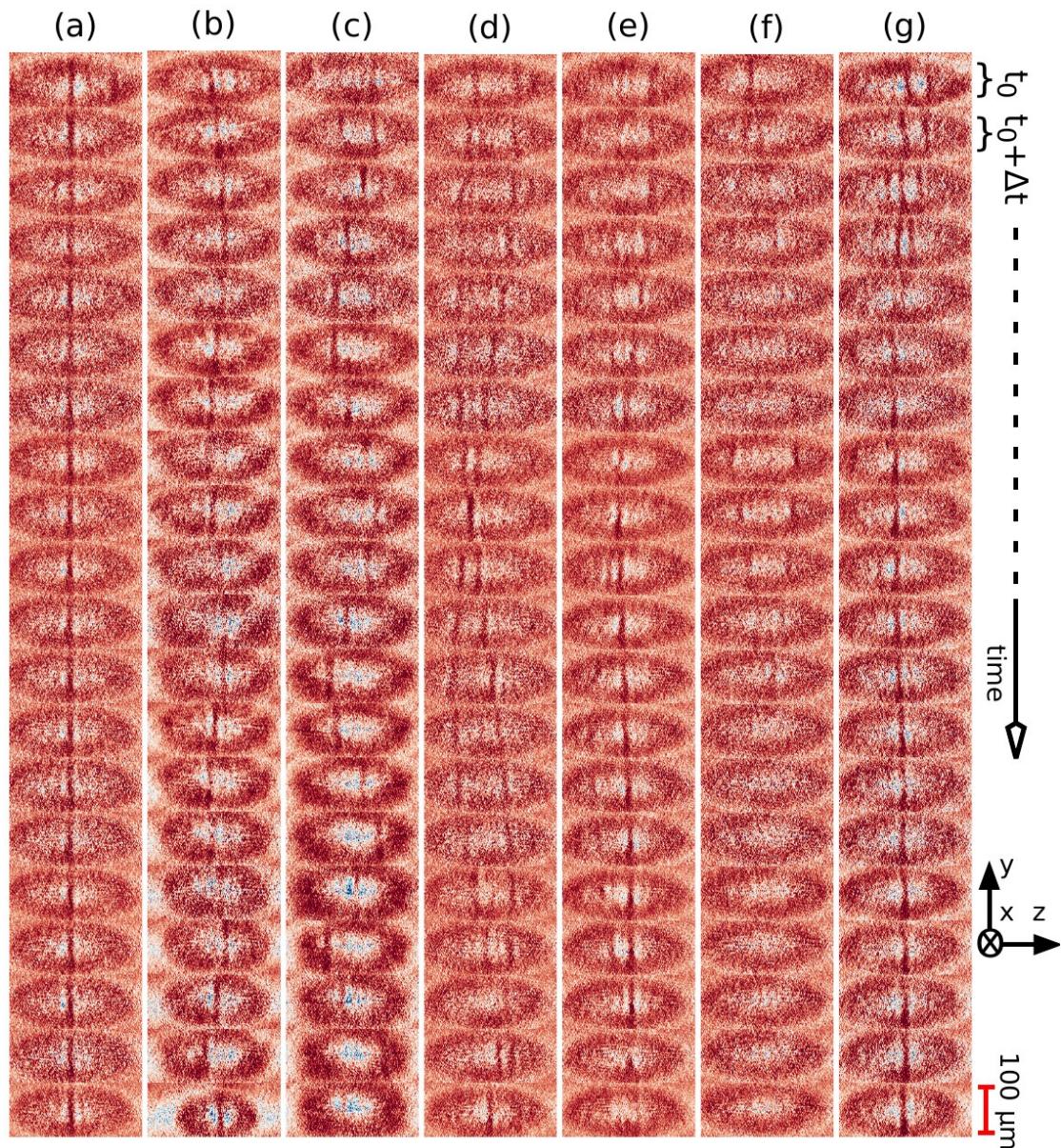
$$|1, -1\rangle \longrightarrow |2, -2\rangle$$

$$\frac{\Delta N}{N_0} \sim 4\%$$

- Initial atom number  $\sim 10^7$
- Magnetic harmonic trap in  $|1, -1\rangle$  with  $\{\omega_{x,y} = \omega_\perp, \omega_z\}/2\pi = \{131, 13\}$  Hz
- 13 ms expansion in  $|2, -2\rangle$  plus RF dressing
- Selective imaging of the output coupled fraction

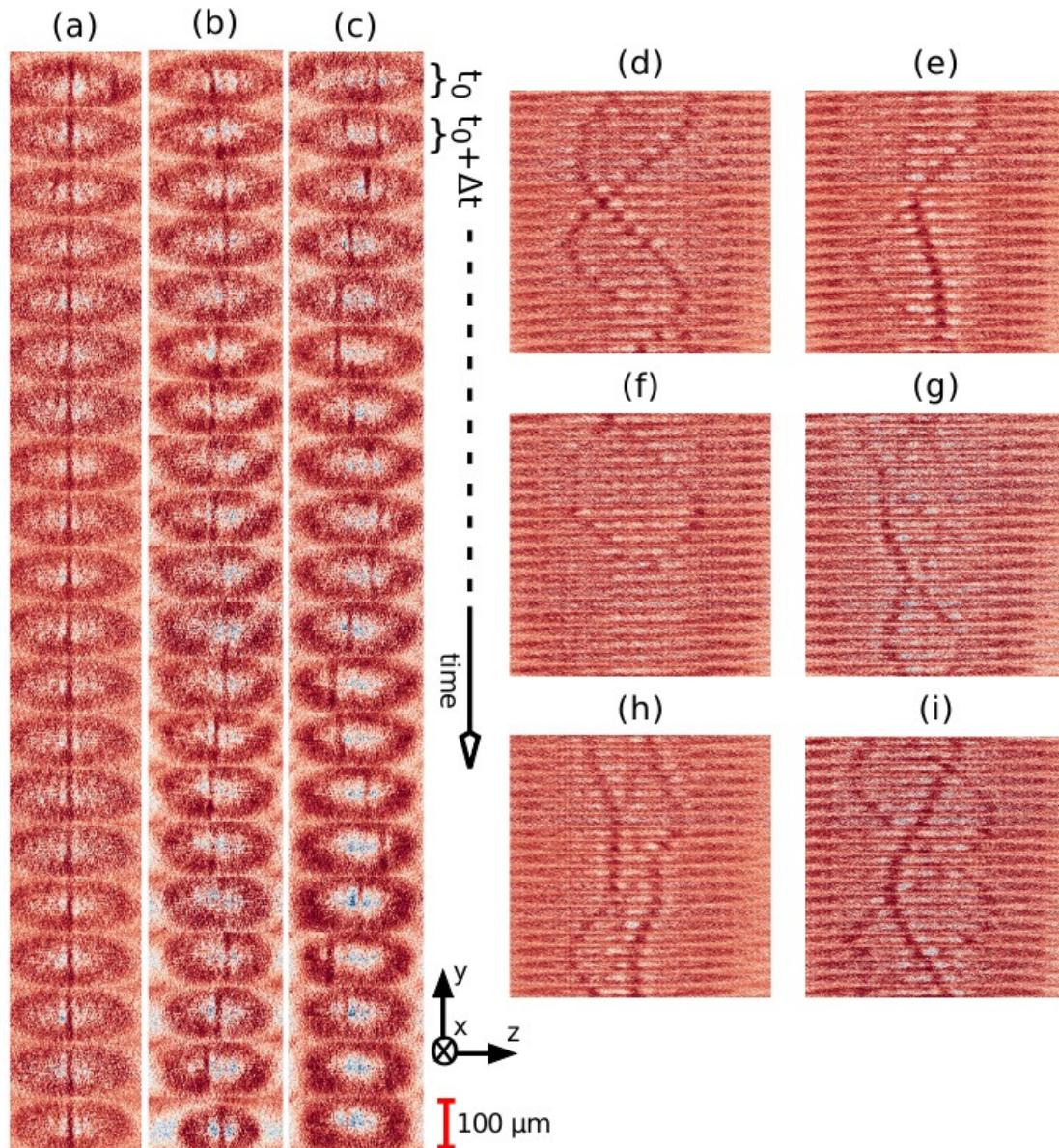


# Stroboscopic imaging of defect dynamics



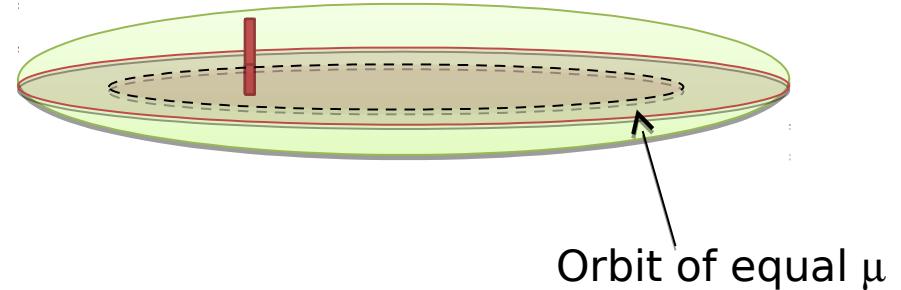
- expansion in the anti-trapped state
- selective imaging of the output coupled fraction
- imaging iterated up to 20 times
- plot residuals from the Thomas-Fermi profile

# Stroboscopic imaging of defect dynamics



- expansion in the anti-trapped state
- selective imaging of the output coupled fraction
- imaging iterated up to 20 times
- plot residuals from the Thomas-Fermi profile

# Vortex dynamics



## SINGLE VORTEX DYNAMICS

A straight vortex line is expected to precess in an inhomogeneous non-rotating condensate, following an equipotential elliptical orbit around the center:

$$T_{SV} = \frac{4(1 - r_0^2)\mu}{3\hbar\omega_\perp \ln(R_\perp/\xi)} T_z$$

$$T_z = \frac{2\pi}{\omega_z} \quad \text{axial trapping period}$$

$$r_0 = \frac{z_{\max}}{R_z} = \frac{y_{\max}}{R_\perp} \quad \text{normalized oscillation amplitude}$$

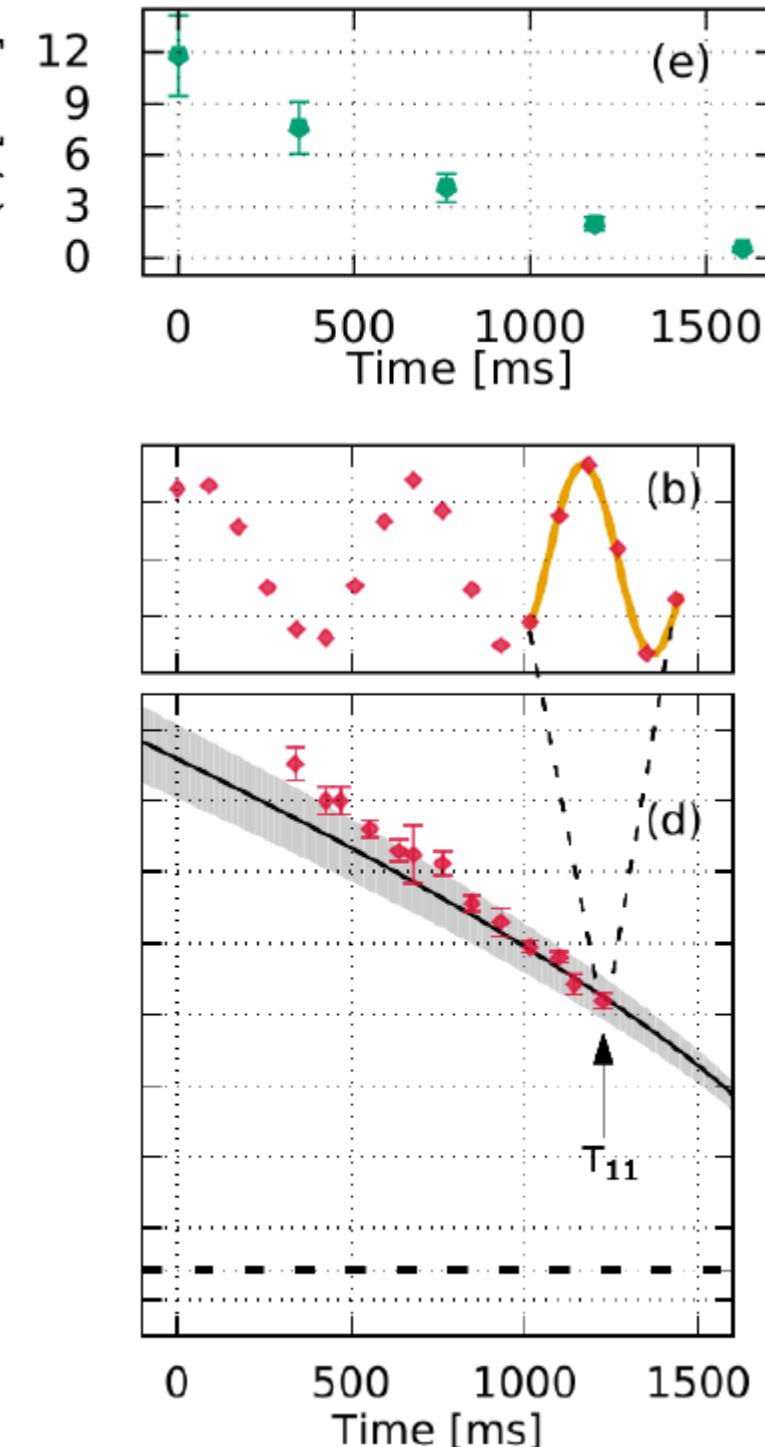
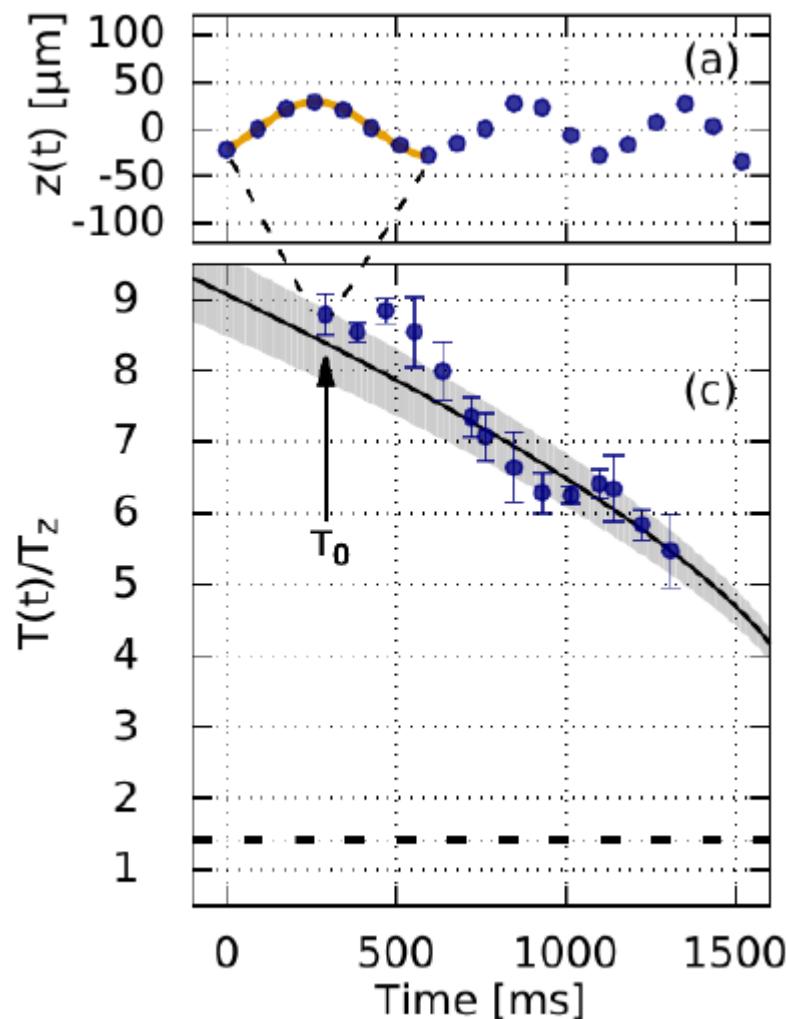
$$\xi \quad \text{condensate healing length}$$

A. L. Fetter and J.-K. Kim, J. Low Temp. Phys. **125**, 239 (2001)

L. P. Pitaevskii, arXiv: 1311.4693 (2013), M. J. H. Ku *et al.*, PRL **113**, 065301 (2014)

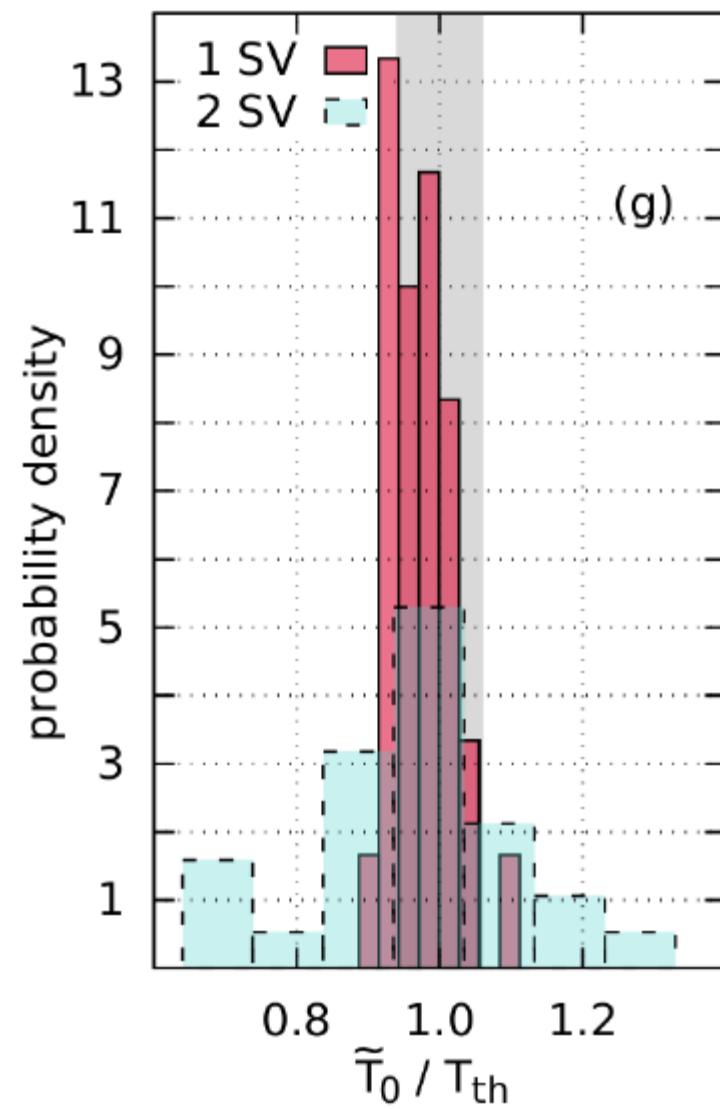
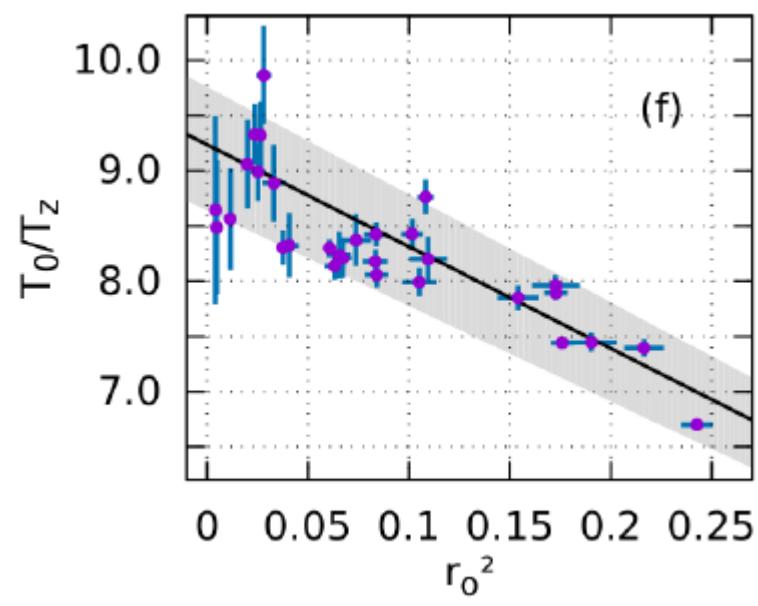
# Period VS atom number

$$T_{SV} \propto \mu \propto N(t)^{2/5}$$



## Period VS amplitude of orbit

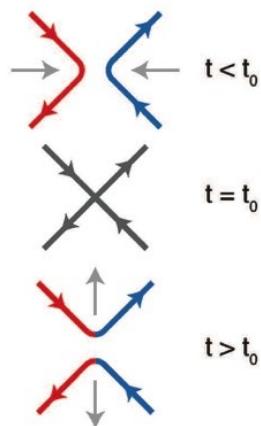
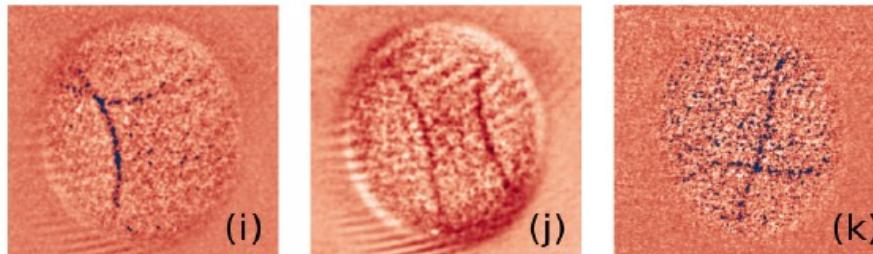
$$\frac{T_{SV}}{T_z} = \frac{4(1 - r_0^2)\mu}{3\hbar\omega_\perp \ln(R_\perp/\xi)}$$



# Interaction among vortices

Random orientation of the nodal lines in the radial plane

Full 3D vortex interaction



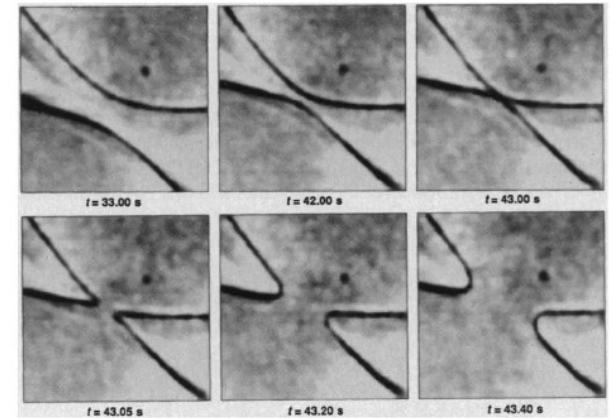
Ideal benchmark for:

- Vortex annihilation
- Vortex decay
- Vortex reconnection

Present simulations:  
Vortices are initially  
at rest

Our experiment:  
finite relative momentum

Reconnection in liquid crystals

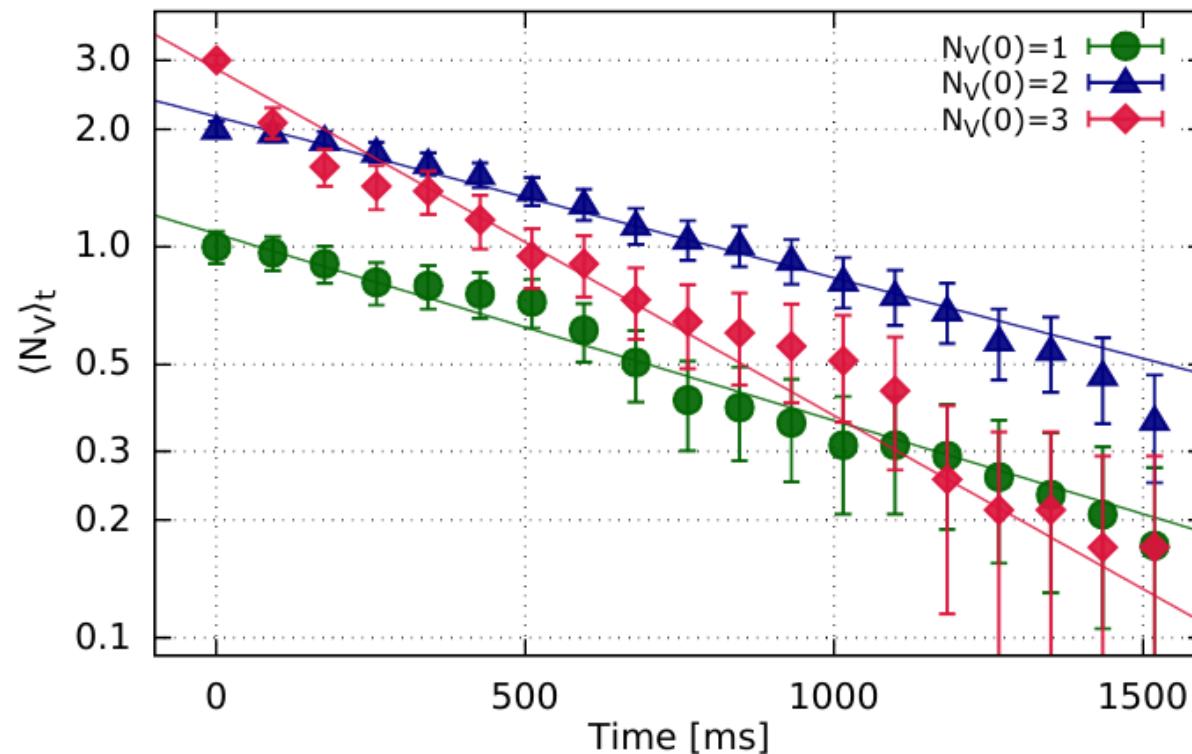


Chuang et al., Science **251**, 1336 (1991)

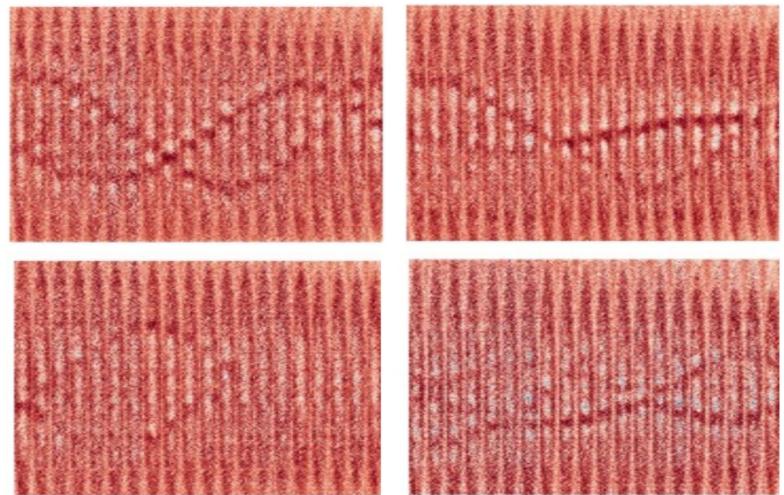
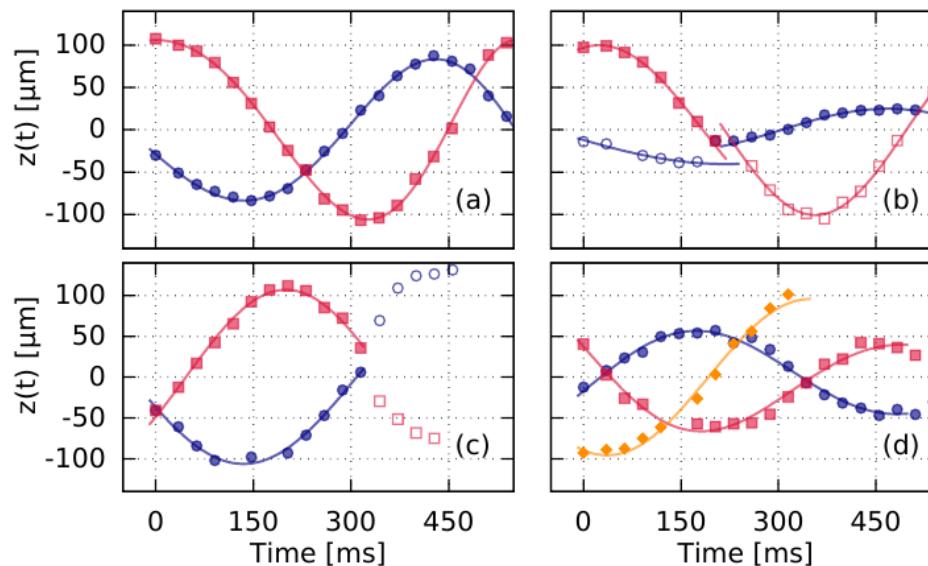
# Interaction among vortices: lifetime measurement

1 or 2 vortices: decay by dissipation with the thermal fraction

3 vortices: faster decay



# Interaction among vortices: phase delays

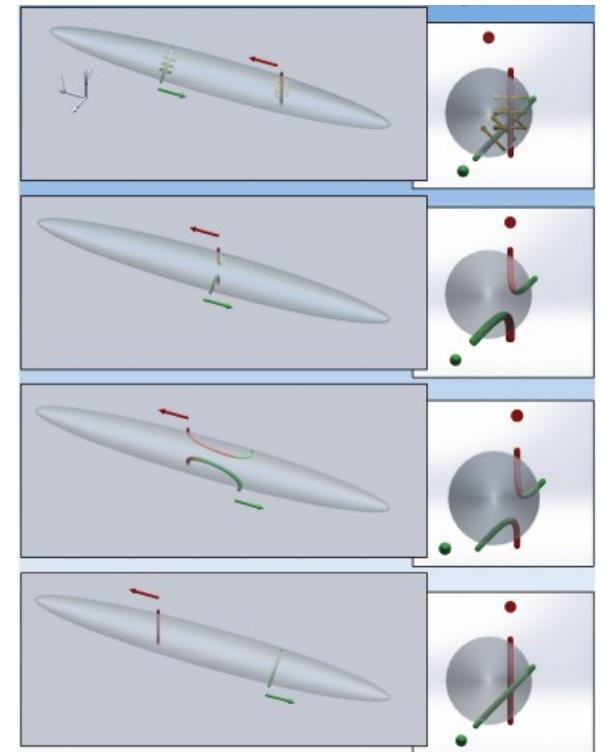


- Frequently: no visible interactions
- Frequently: change of visibility
- Sometimes: phase shifts
- Seldom: annihilations

Single reconnection energetically expensive due to nodal line stretching.

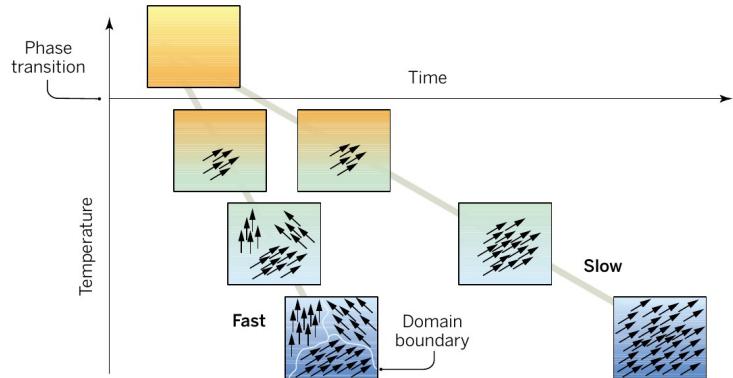
Possible alternatives:

- double reconnection
- rotation of the nodal lines when approaching

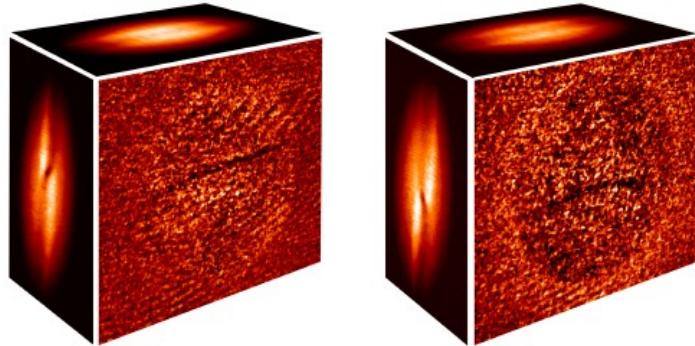


# Summary

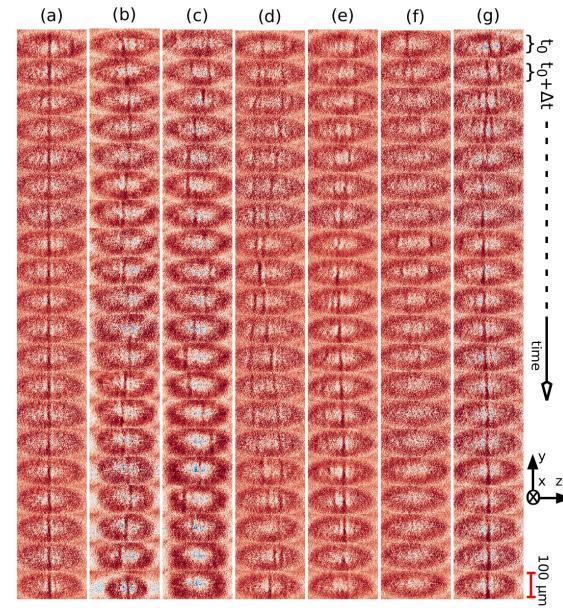
## formation



## nature



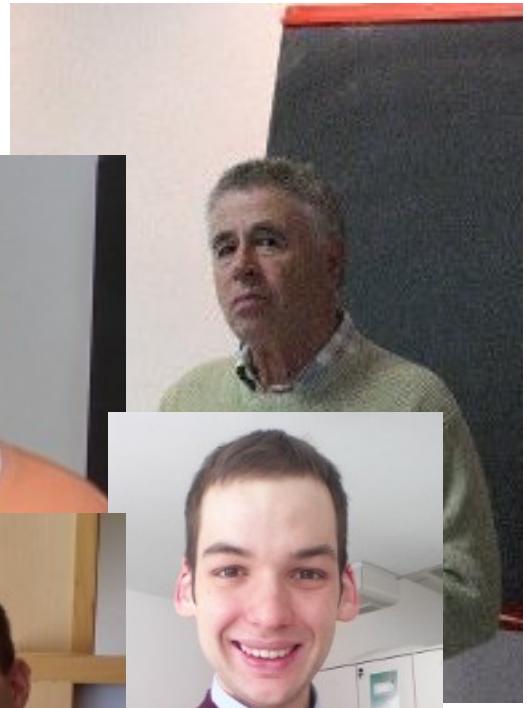
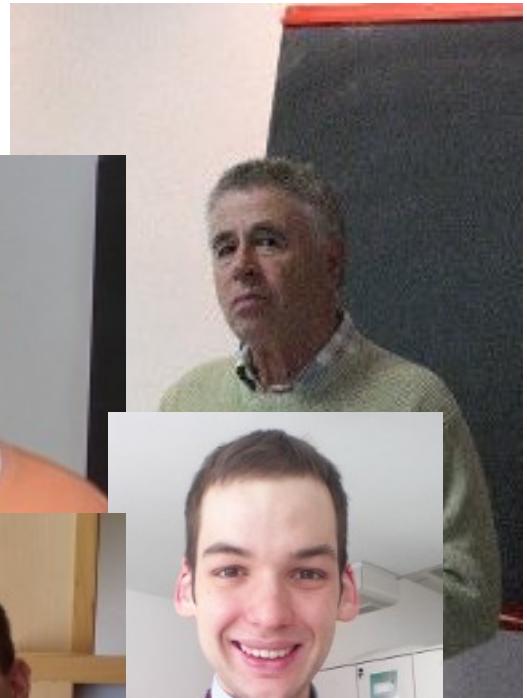
## dynamics & interaction



## future developments

- investigation of post-quench dynamics after crossing phase transitions
- microscopic study of reconnection mechanisms

# Thank you!



Simone Serafini  
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