Many-Body Phyics with Cavity Quantum Electrodynamics

Giovanna Morigi
Universität des Saarlandes

Why cavity QED?

Atom-photon interactions at the limit

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For understanding the interplay between noise and interactions in the quantum world

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Atom-photon interactions at the limit

→ For understanding the interplay between noise and interactions in the quantum world

→ For photonic quantum simulators

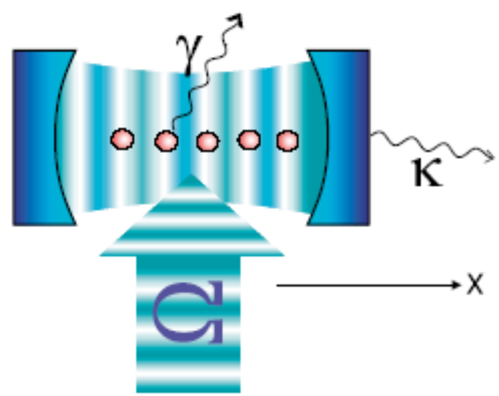
Outline

- Long-range-interaction physics in many-body cavity QED

- Long-range-interaction physics in presence of frustration

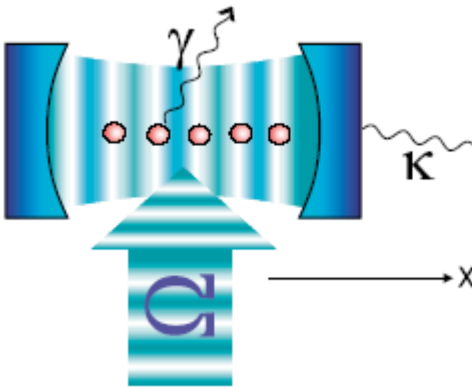
Quantum structures in cavity QED

Originate from the mechanical effects of light in a high-finesse cavity



Atoms in an optical cavity

- Atoms driven far-off resonance: coherent scattering into the cavity mode - classical dipoles
- Atoms move (quantum motion): dynamical refractive index

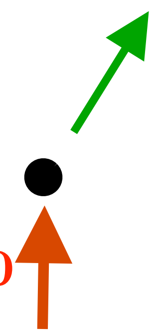


$$\hat{\mathcal{H}}_{\mathrm{eff}} = \sum_{j=1}^{N} \frac{\hat{p}_j^2}{2m_j} - \hbar \left[\Delta_c - \sum_{j=1}^{N} U_j \cos^2(k\hat{x}_j) \right] \hat{a}^\dagger \hat{a} + \hbar \sum_{j=1}^{N} S_j \cos(k\hat{x}_j)(\hat{a} + \hat{a}^\dagger).$$

- Quantum structures: emerge from the interplay between coherent scattering and photon losses

Mechanical effects of light

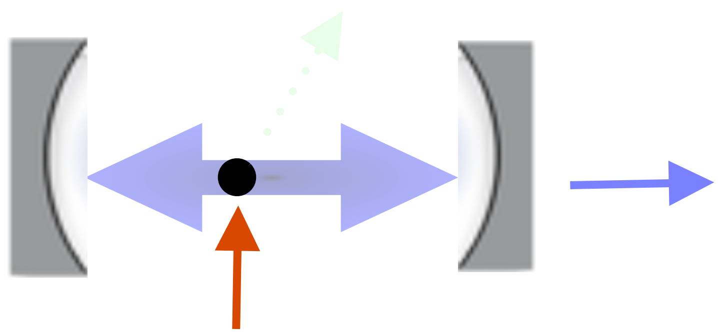
Spontaneous emission: $h\omega$



Laser photon: $h\omega$

ω < ω: energy is transferred from the atom center of mass into the electromagnetic field.

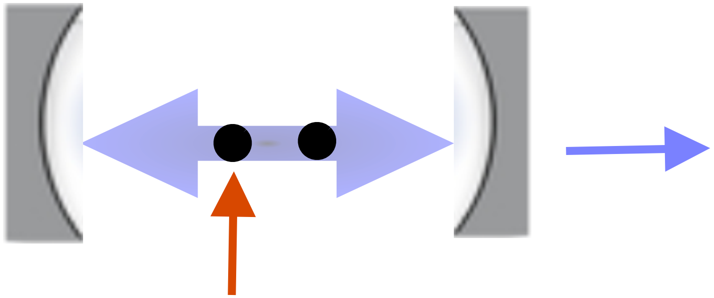
Mechanical effects of light in a cavity



atom coherently scatter into the cavity field
The phase of the emitted light depends on the atom position in the cavity mode

ω < ω: (cavity) cooling

Photon-mediated interactions



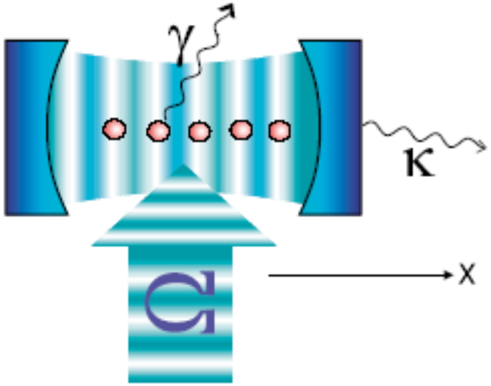
The phase of the emitted light depends on
the atomic positions in the cavity

The cavity field mediates an effective interaction

Photon-mediated interactions are long-range forces

In a single-mode resonator the electric field is coherent over the whole atomic ensemble

The cavity-mediated interaction belongs to the class of long-range potentials $1/r^{a}$ with exponent a < dimension d (e.g.: Gravitation and Coulomb at $d>1$)



Statistical mechanics with long-range potentials

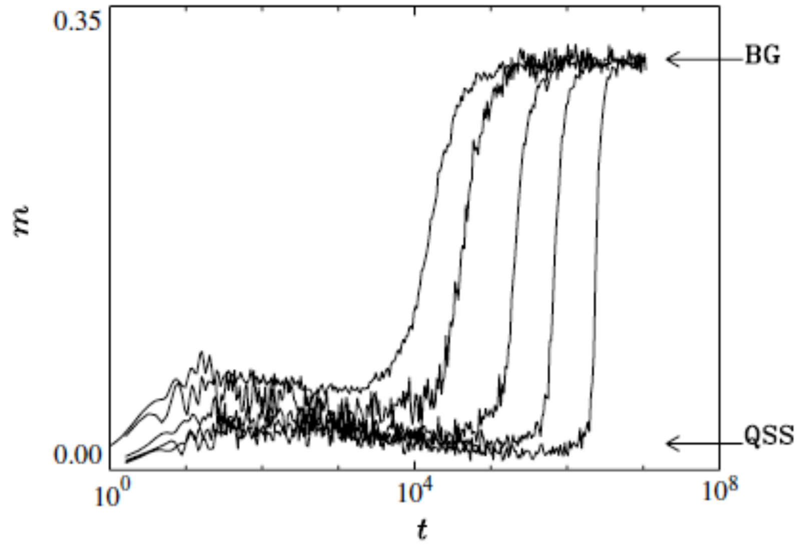
Non-additivity: the energy of a system is not the sum of the energies of the partitions (not even in the thermodynamic limit)

Ensembles are in general not equivalent (revisit phase transitions....)

Dynamics exhibit prethermalization over diverging time scales (quasi-stationary states)

see e.g.: A. Campa, T. Dauxois, S. Ruffo, Phys. Rep. 480, 57 (2009)

Quasi-stationary states



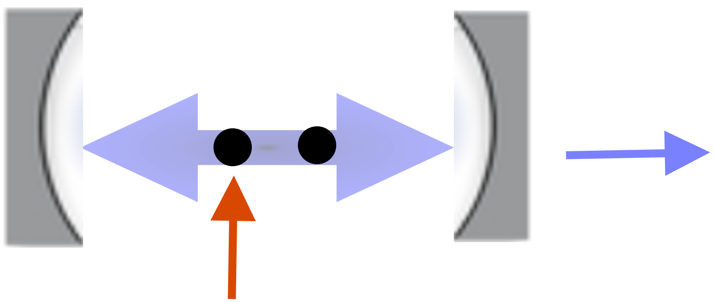
$$N = 10^3, 2 \times 10^3, 5 \times 10^3, 10^4 \text{ and } 2 \times 10^4$$

Lifetime of QSS increases with N^{1+b}

A. Campa, T. Dauxois, S. Ruffo, Phys. Rep. 480, 57 (2009)

Photon-mediated interactions depend on the pump intensity

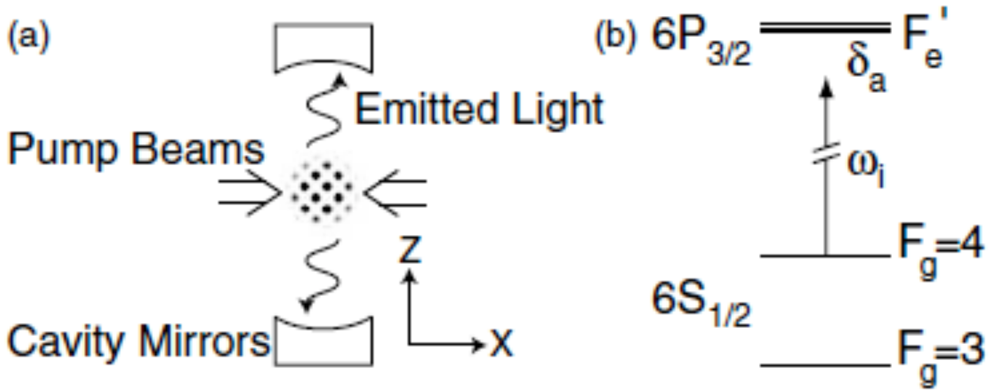
Correlations can form when the field is sufficiently strong

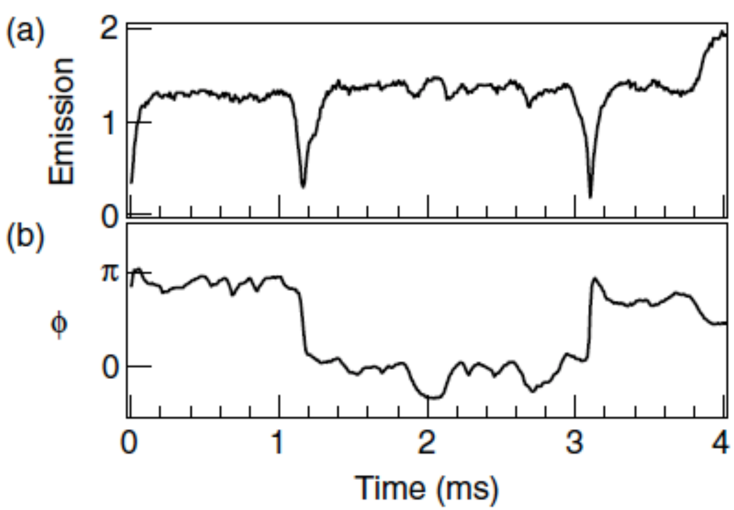


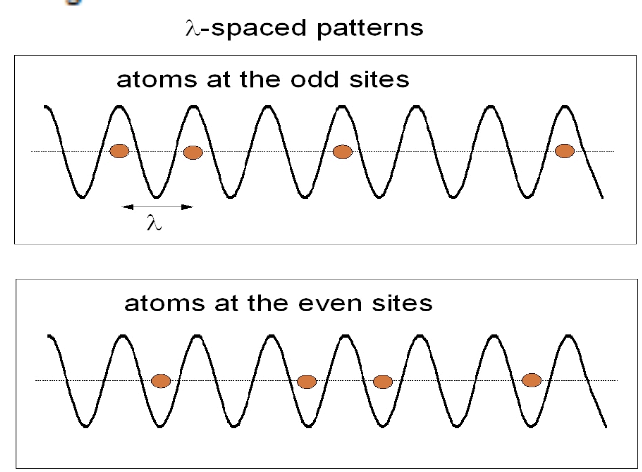
Interplay between pump and losses

Dynamics and phase transitions
are intrinsically out-of-equilibrium

Selforganization of laser cooled atoms



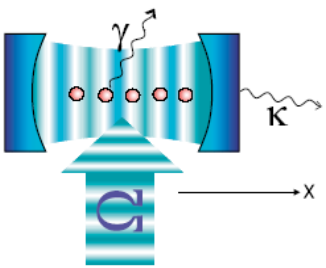




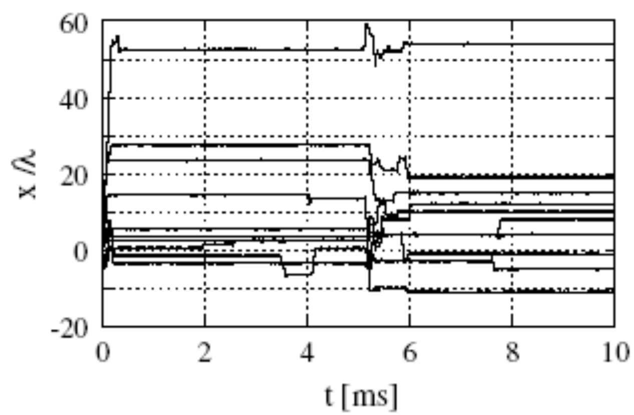
A. T. Black, H. W. Chan, and V. Vuletić, Phys. Rev. Lett. 91, 203001 (2003)

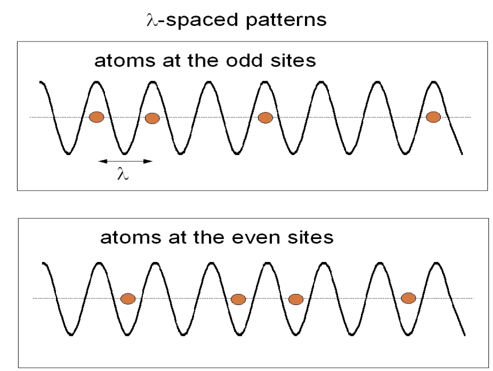
Selforganization in optical cavities

Localization of atomic positions inside the cavity mode



$$\Theta = \sum_{j=1}^{N} \cos(kx_j)/N$$





Atomic pattern: atoms scatter in phase into the cavity mode
The cavity field is maximum and stably traps the atoms

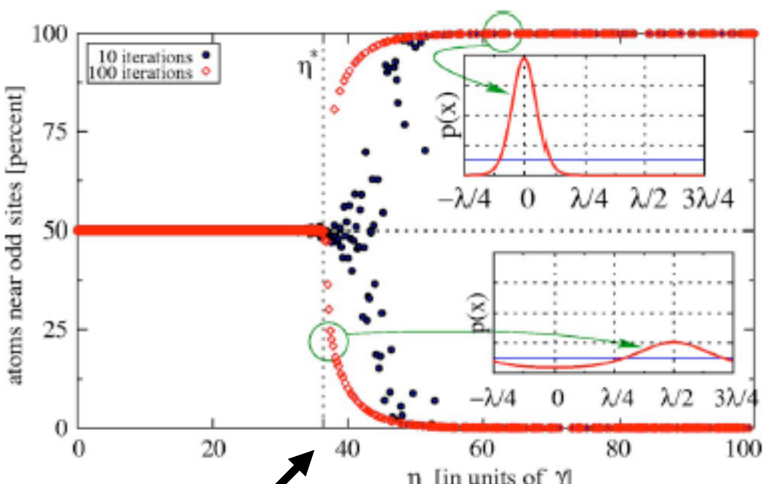
P. Domokos, H. Ritsch, Phys. Rev. Lett. 89, 253003 (2002)

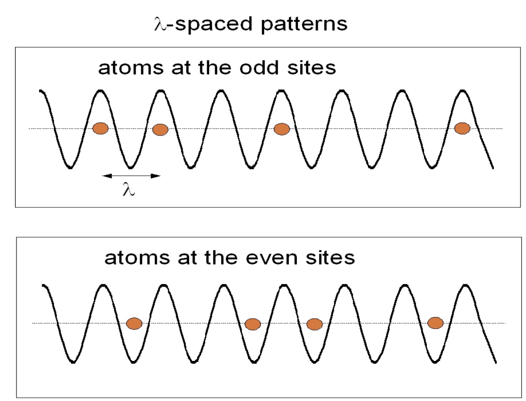
Selforganization in optical cavities

Localization of atomic positions inside the cavity mode

$$\Theta = \sum_{j=1}^{N} \cos(kx_j)/N$$

Bifurcation at threshold:





Pump threshold

J. Asbóth, P. Domokos, H. Ritsch, and A. Vukics, Phys. Rev. A 72 053417 (2005)

Dynamics in the semiclassical regime

- Cavity field is quantum

- Time scale separation of cavity field and external motion

- Wigner function of atoms (field density matrix)

$$\tilde{W}_t(\boldsymbol{x},\boldsymbol{p}) = \tilde{f}(\boldsymbol{x},\boldsymbol{p},t)\sigma_S(\boldsymbol{x}) + \tilde{\chi}(\boldsymbol{x},\boldsymbol{p},t)$$

the field follows adiabatically the motion — non-adiabatic contribution

- Perturbative expansion in
  recoil momentum + retardation effects

J. Dalibard and C. Cohen-Tannoudji, J. Phys. B 18, 1661 (1985).
S. Schütz, H. Habibian, GM, Phys. Rev. A 88, 033427 (2013)

Eliminating the cavity field: Fokker-Planck equation

Motion semiclassical / Cavity field is quantum
retardation effects as perturbations

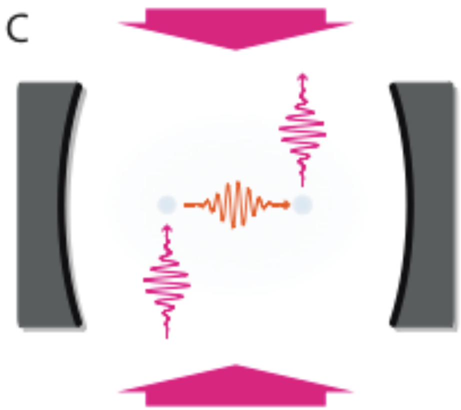
$$f(x_1, p_1; \ldots; x_N, p_N; t)$$

$$\partial_t f + \{f, H\} \simeq$$
$$-\bar{n}\Gamma \sum_i \sin(kx_i)\partial_{p_i} \frac{1}{N} \sum_j \sin(kx_j) \left(p_j + \frac{m}{\beta}\partial_{p_j}\right) f$$

S. Schütz, H. Habibian, GM, Phys. Rev. A 88, 033427 (2013)

Hamiltonian dynamics

Photons mediate long-range forces between the atoms



R. Mottl, PhD thesis

Effective Hamiltonian

$$H = \sum_j \frac{p_j^2}{2m} + \hbar\Delta_c \bar{n} N \Theta^2 + \mathrm{O}(U)$$

$$\Theta = \sum_{j=1}^{N} \cos(kx_j)/N$$

Infinitely long-range interactions

Analogy with Hamiltonian-Mean-Field Model (HMF)

see e.g.: A. Campa, T. Dauxois, S. Ruffo, Phys. Rep. 480, 57 (2009)

Noise also establishes long-range correlations

$$\partial_t f + \{f, H\} \simeq$$

$$- \bar{n}\Gamma \sum_i \sin(kx_i)\partial_{p_i} \frac{1}{N} \sum_j \sin(kx_j)\left(p_j + \frac{m}{\beta}\partial_{p_j}\right) f$$

Gratings at the minima of the cos-potential are “dark”

Steady state I

$$\partial_t f_\infty = 0$$

$$\partial_t f + \{f, H\} \simeq$$
$$-\bar{n}\Gamma \sum_i \sin(kx_i)\partial_{p_i} \frac{1}{N} \sum_j \sin(kx_j) \left(p_j + \frac{m}{\beta}\partial_{p_j}\right) f$$

Steady state is a thermal distribution

$$f_\infty = f_0 \exp(-\beta H)$$

The temperature is tuned by the laser frequency

$$\hbar\beta = -4\Delta_c/(\Delta_c^2 + \kappa^2)$$

An ensemble is cooled like a single atom....

S. Schütz, GM, Phys. Rev. Lett. 113, 203002 (2014)

Steady state II

$$\partial_t f_\infty = 0$$

$$f_\infty = f_0 \exp(-\beta H)$$

Cross-correlations are important for large photon numbers

$$H = \sum_j \frac{p_j^2}{2m} + \hbar\Delta_c \bar{n} N \Theta^2 + \mathrm{O}(U)$$

S. Schütz, GM, Phys. Rev. Lett. 113, 203002 (2014)

Steady state magnetization

$$f_\infty = f_0 \exp(-\beta H)$$

Free energy per particle

$$\mathcal{F}(\Theta) \approx \frac{1}{\beta}\left[\left(1 - \frac{\bar{n}}{\bar{n}_c}\right)\Theta^2 + \frac{5}{4}\Theta^4\right]$$

Selforganization Threshold:

$$\bar{n}_c = \frac{\kappa^2 + \Delta_c^2}{4\Delta_c^2}$$

S. Schütz, S. Jäger, GM, preprint (2015)

Steady state magnetization

$$f_\infty = f_0 \exp(-\beta H)$$

Selforganization Threshold:

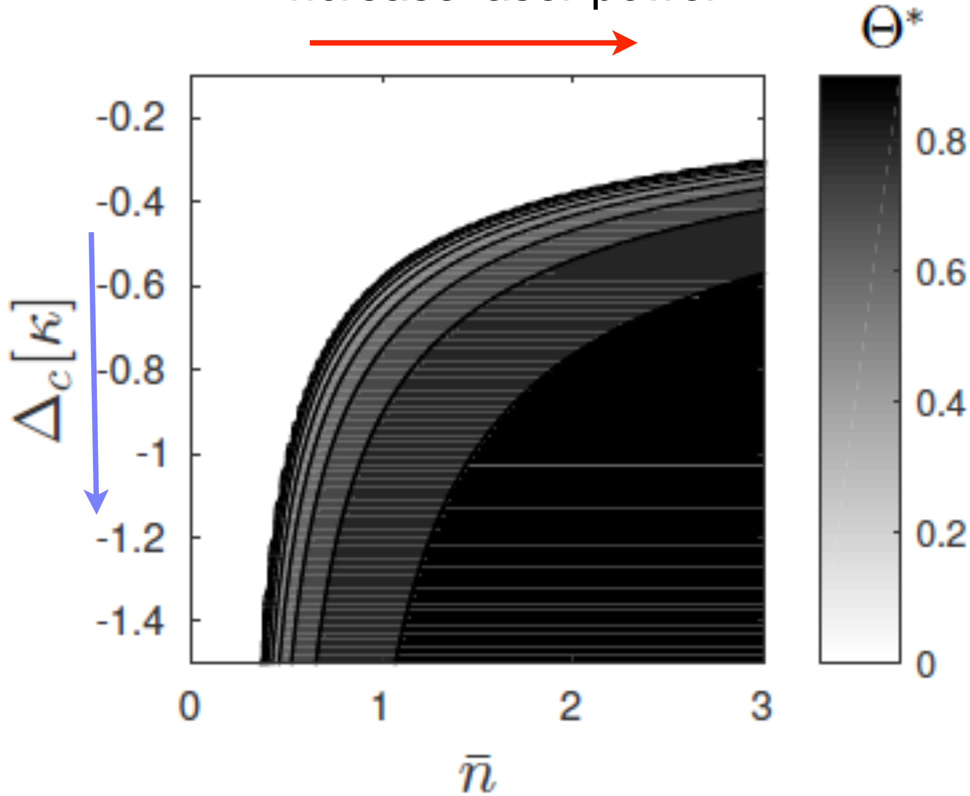
$$\bar{n}_c = \frac{\kappa^2 + \Delta_c^2}{4\Delta_c^2}$$

Temperature:

$$\hbar\beta = -4\Delta_c/(\Delta_c^2 + \kappa^2)$$

change temperature

Increase laser power



S. Schütz, S. Jäger, GM, preprint (2015)

Steady state magnetization

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Selforganization Threshold:

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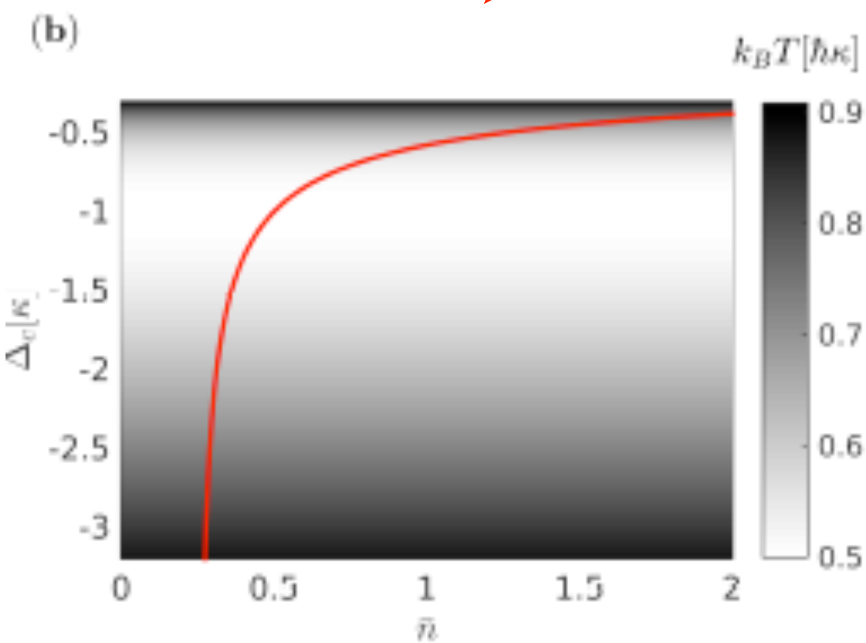
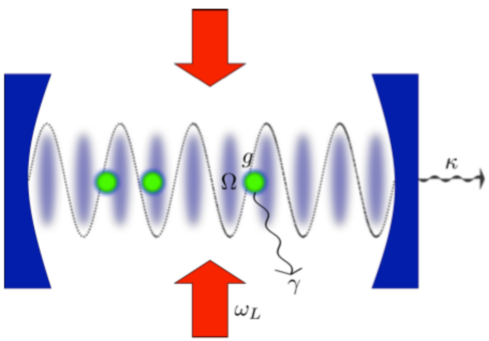
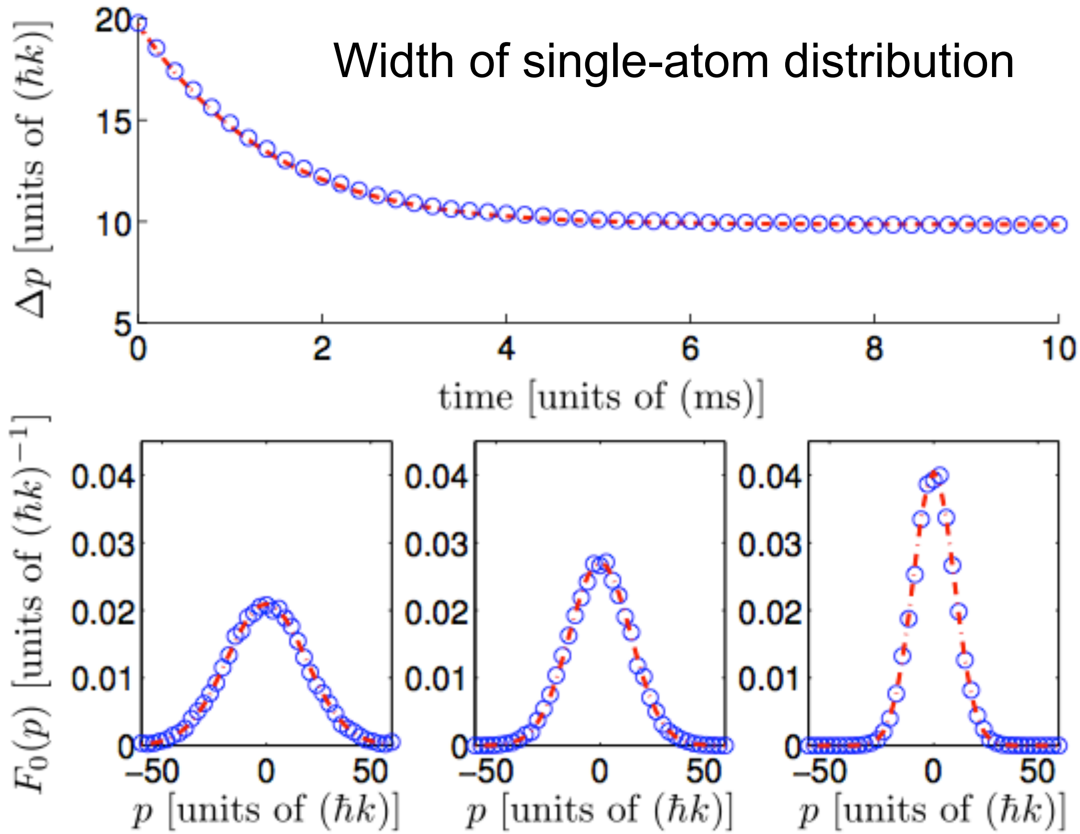


S. Schütz, S. Jäger, GM, preprint (2015)

Dynamics below threshold





$$t = (0.1, 1, 9)\ \text{ms}$$

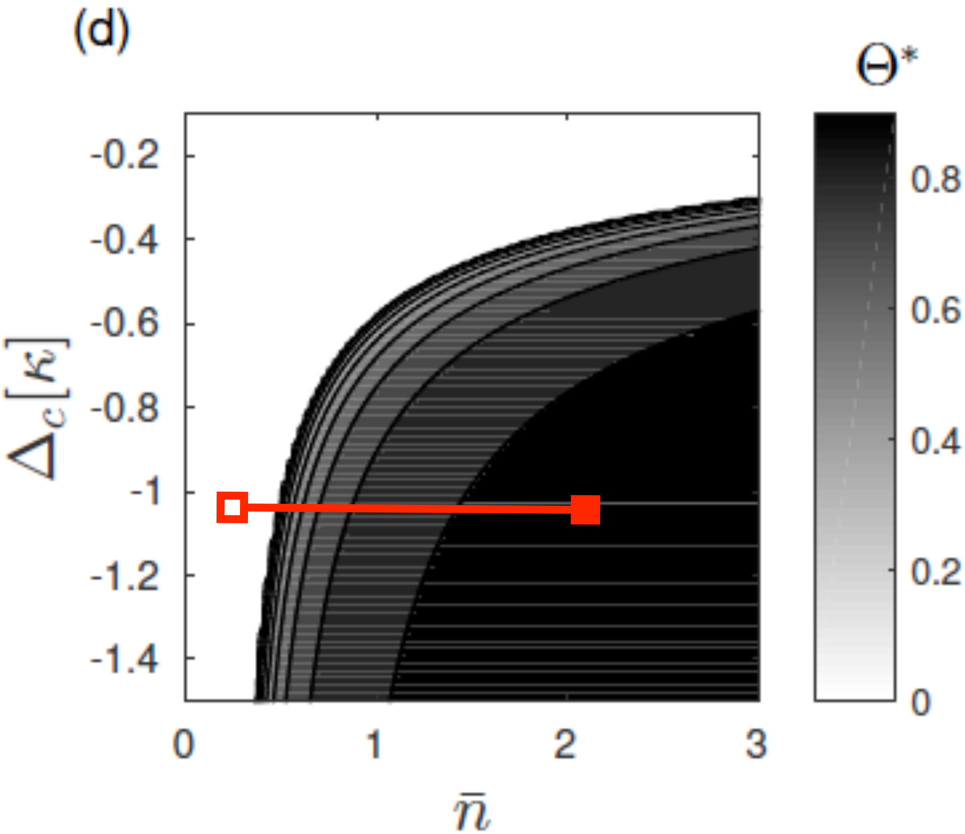
1ms= $10^3\,\kappa^{-1}$

Maxwell-Boltzmann distribution

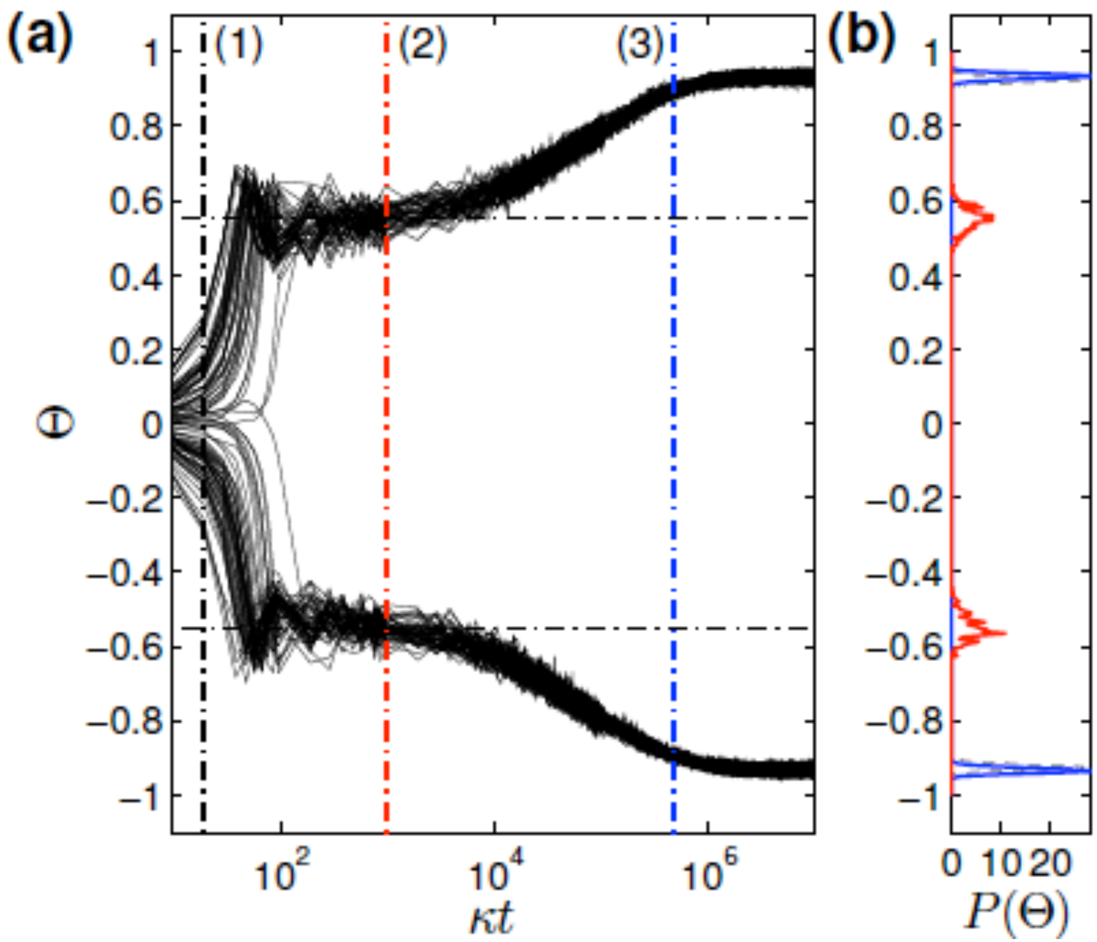
S. Schütz, H. Habibian, GM, Phys. Rev. A 88, 033427 (2013)

Quench across the transition

Sudden quench of the field intensity
from below to above threshold

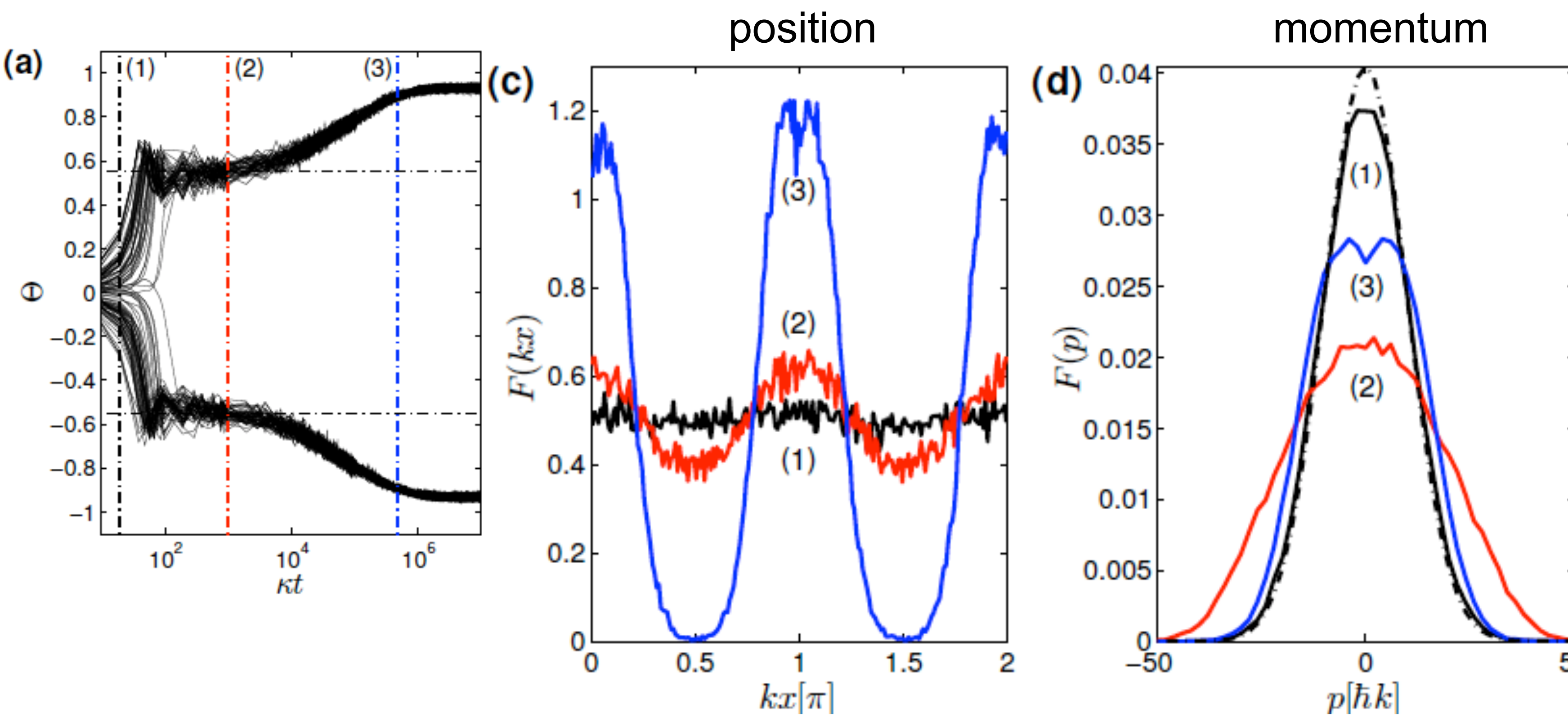


Dynamics above threshold



Long-lived prethermalized state

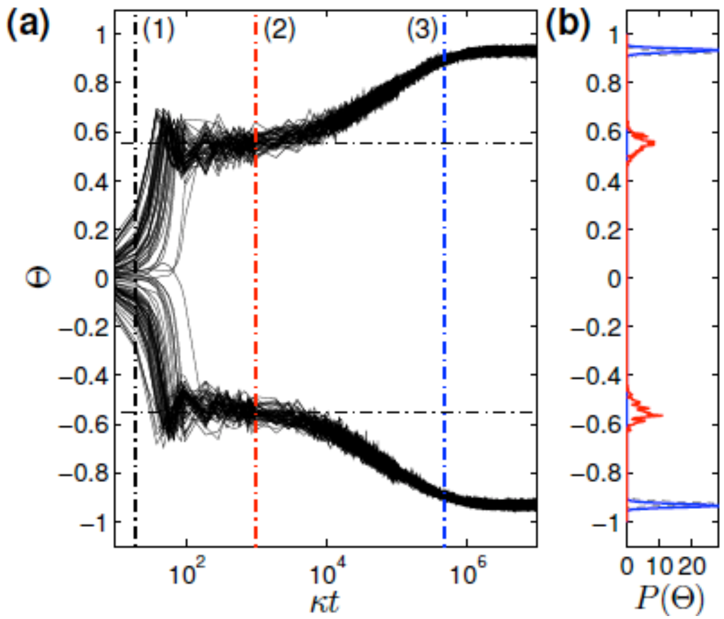
Dynamics above threshold

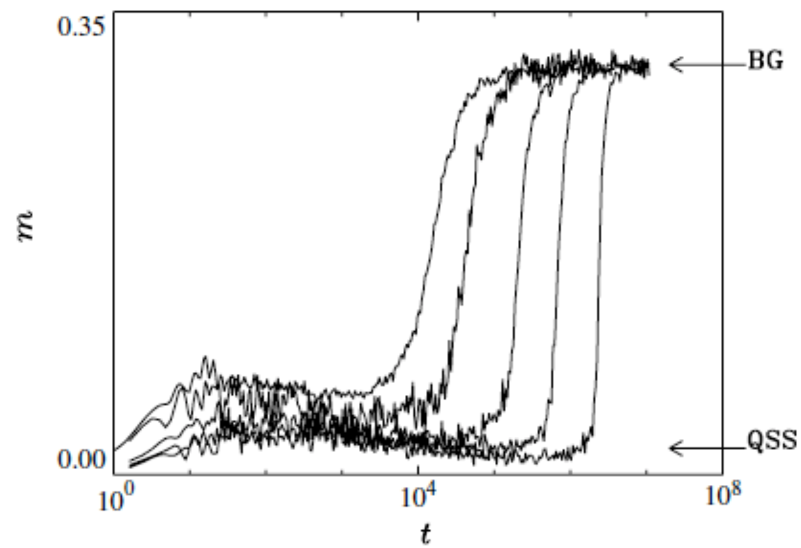


Metastable state is non thermal

S. Schütz, GM, Phys. Rev. Lett. 113, 203002 (2014)

Quasi-stationary state?



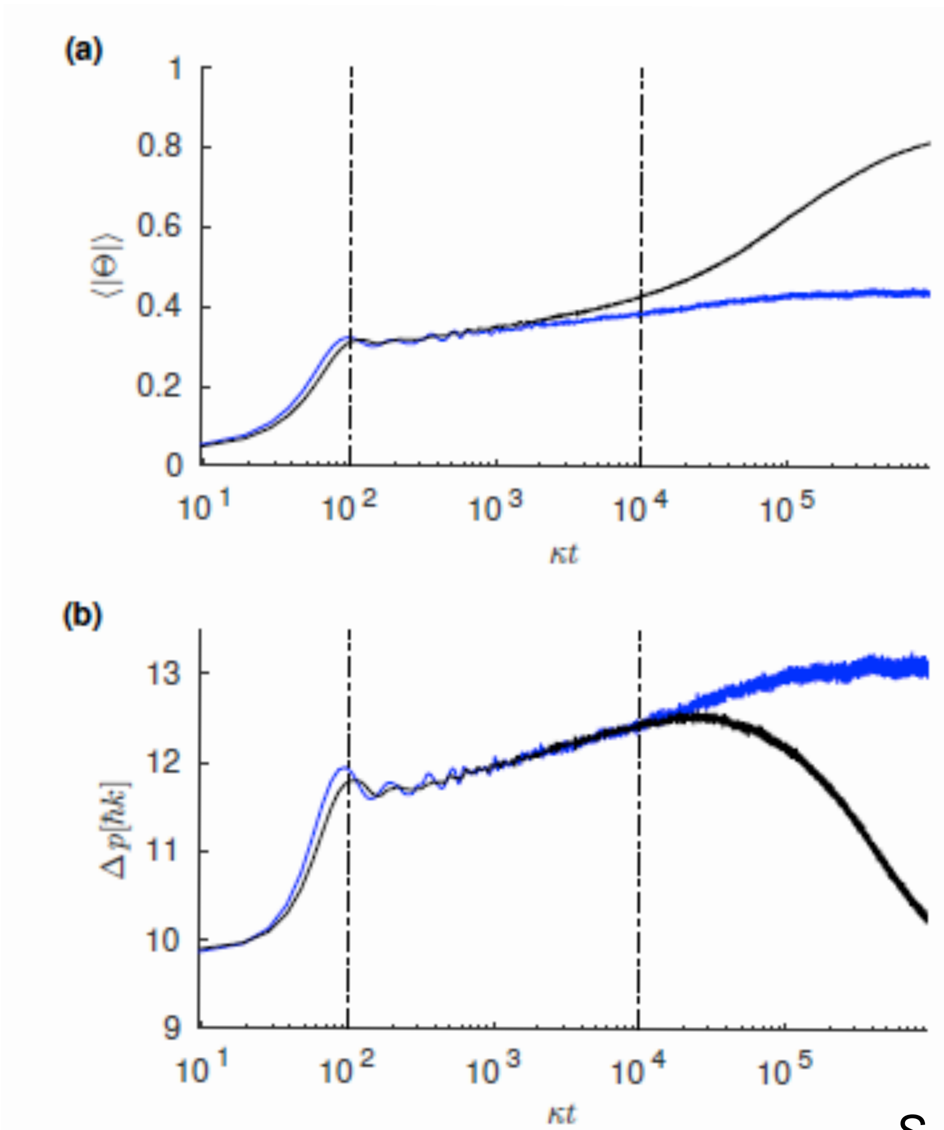


coherent and dissipative dynamics are at the same time scale

noise induces long-range correlations

metastable state is a quasi-dark state

Collisions vs quantum noise

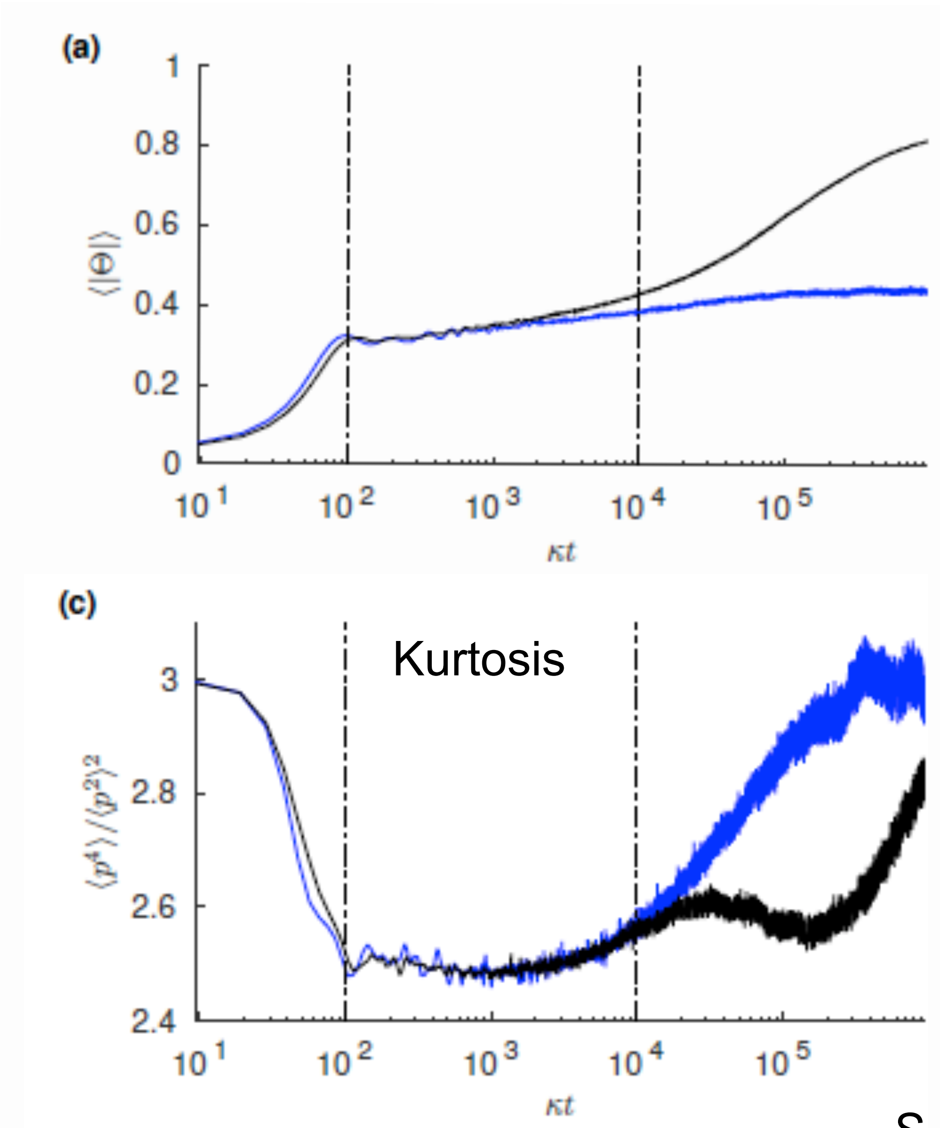


full dynamics

Hamiltonian dynamics

S. Schütz, S. Jäger, GM, preprint (2015)

Collisions vs quantum noise

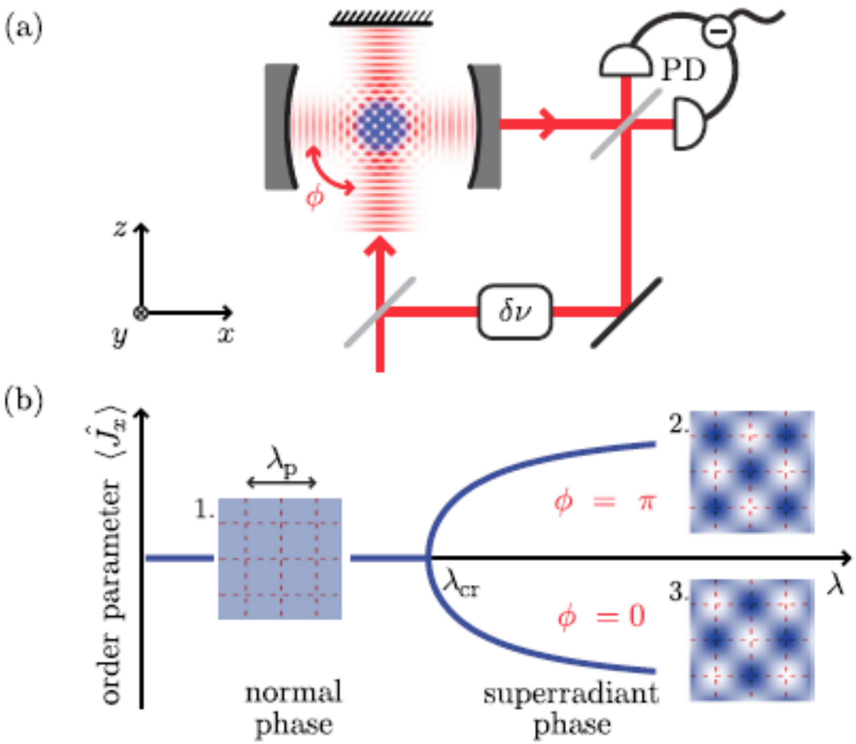


quantum noise is responsible for non-thermal behaviour

S. Schütz, S. Jäger, GM, preprint (2015)

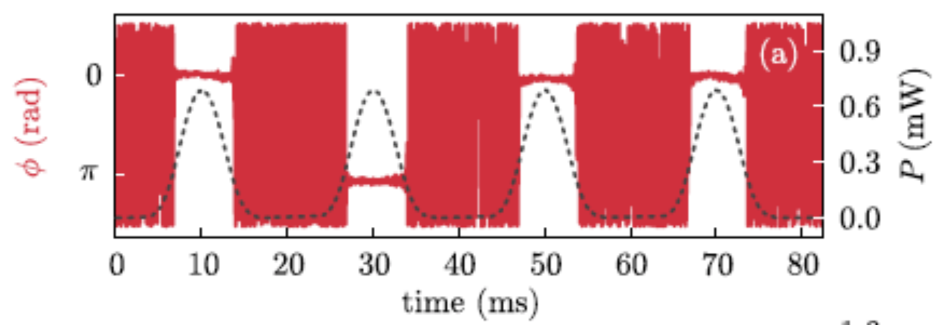
Quantum vs Thermal atoms

Selforganization in the ultracold



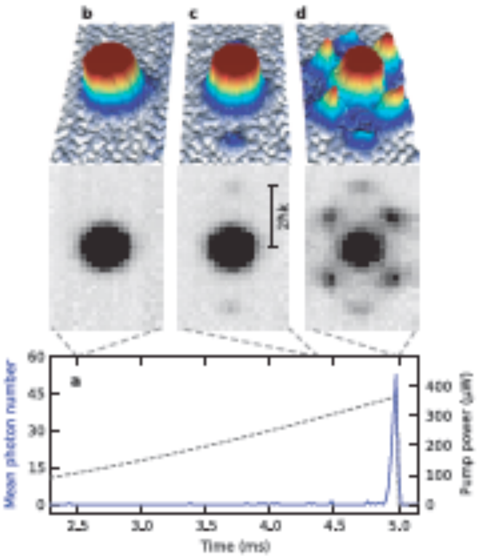
Dispersive regime:
dynamics is conservative

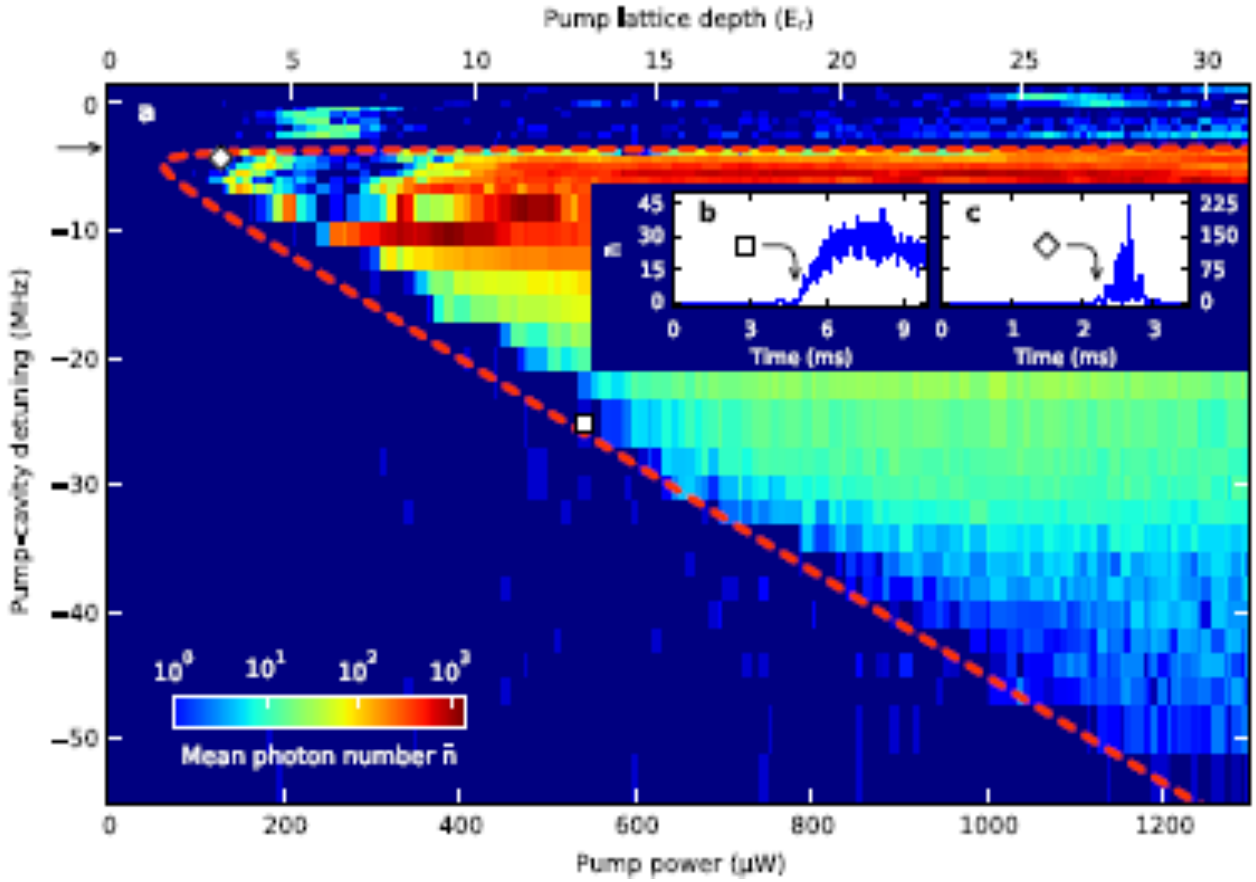
Evidence of the two possible patterns



K. Baumann, R. Mottl, F. Brennecke, and T. Esslinger, Phys. Rev. Lett. 107 140402 (2011)

Dicke phase transition

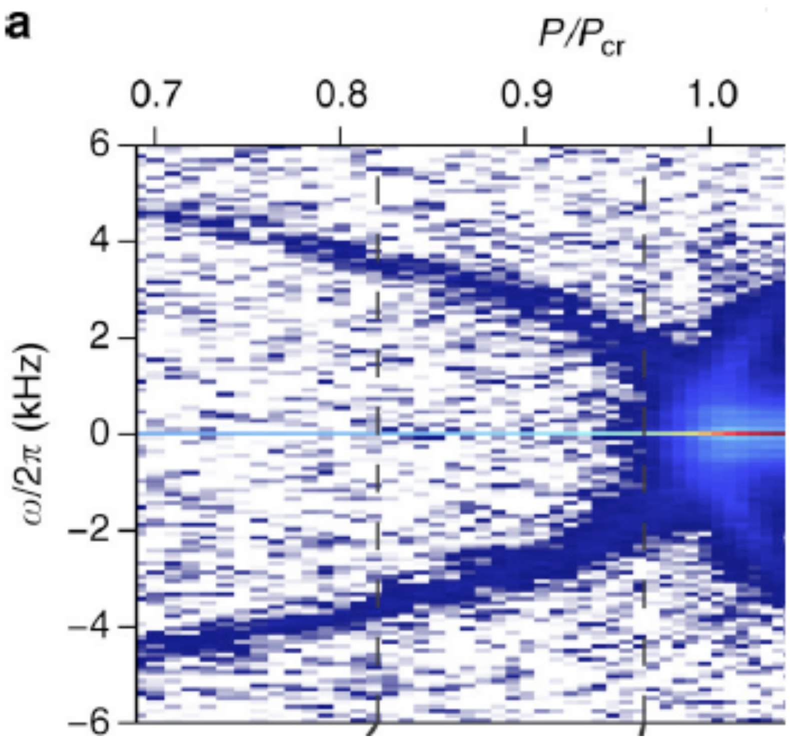




Transition from normal SF to Supersolid phase

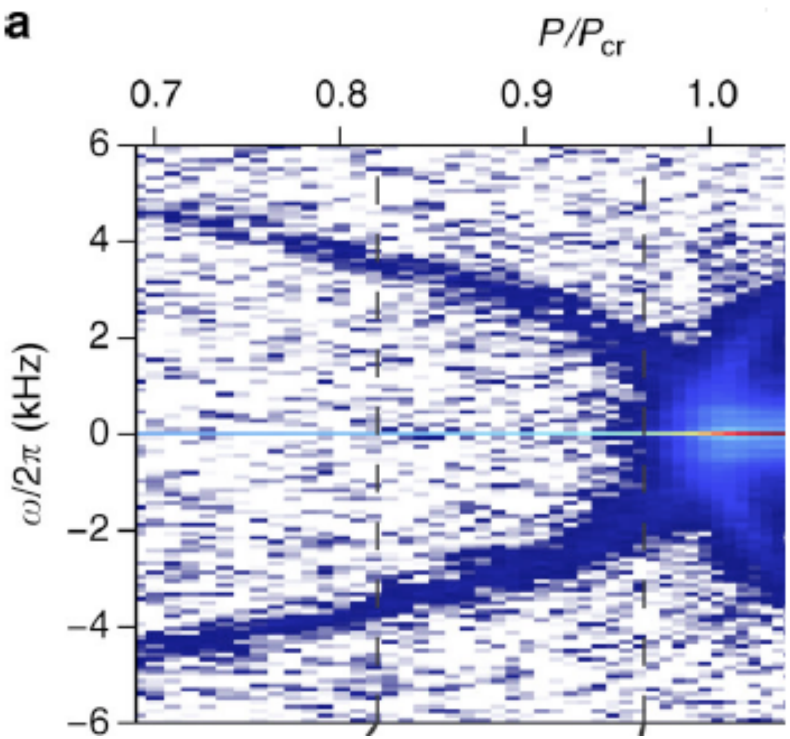
K. Baumann, C. Guerlin, F. Brennecke, T. Esslinger, Nature 464, 1301 (2010)

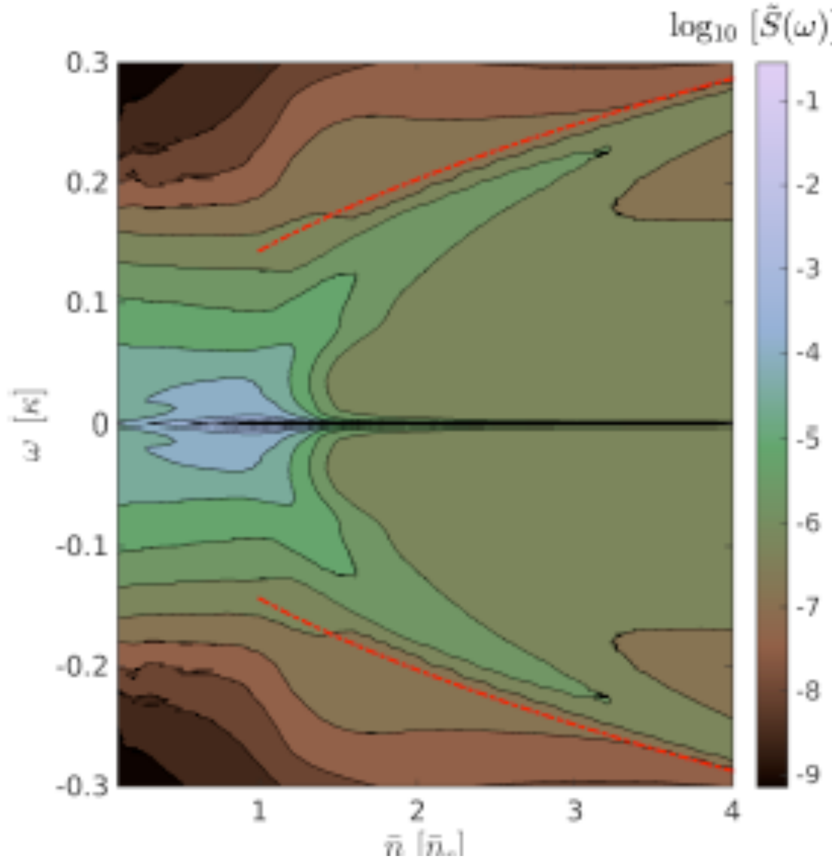
Power spectrum



R. Landig, F. Brennecke, R. Mottl, T. Donner, and T. Esslinger, Nat. Comm. 6, 7046 (2015).

Power spectrum





The semiclassical theory makes good qualitative predictions of the correlation functions of light at the cavity output above threshold

S. Schütz, S. Jäger, GM, preprint (2015)

Quantum vs Thermal atoms

- Selforganization is due to Bragg gratings: the properties of light depend on atomic density

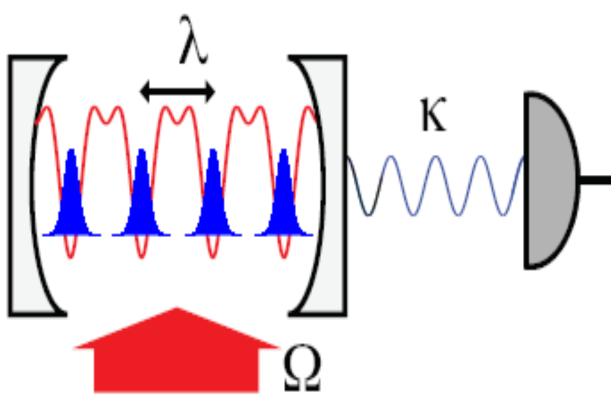
- It does not destroy the quantum phase of matter in the Hamiltonian regime (superfluid->supersolid)

- Are there quantum phase transition induced by the cavity?

Short vs Long range

$$H = \sum_j \frac{p_j^2}{2m} + \hbar\Delta_c \bar{n} N \Theta^2 + \text{s-wave scattering}$$

The atoms are trapped in the potential they scatter:
the coefficients of the Bose-Hubbard depend on density



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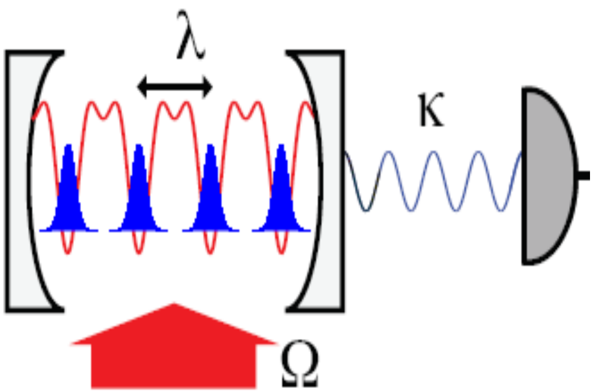


$$\hat{\mathcal{H}}_{\mathrm{BH}}^{(1D)} = -\sum_i t\,(\hat{b}_i^\dagger \hat{b}_{i+1} + \hat{b}_{i+1}^\dagger \hat{b}_i) + \frac{U}{2}\sum_i \hat{n}_i(\hat{n}_i - 1)$$
$$+ \frac{\hbar s_0^2}{\hat{\delta}_{\mathrm{eff}}^2 + \kappa^2} K \hat{\Phi}^2 \hat{\delta}_{\mathrm{eff}}$$

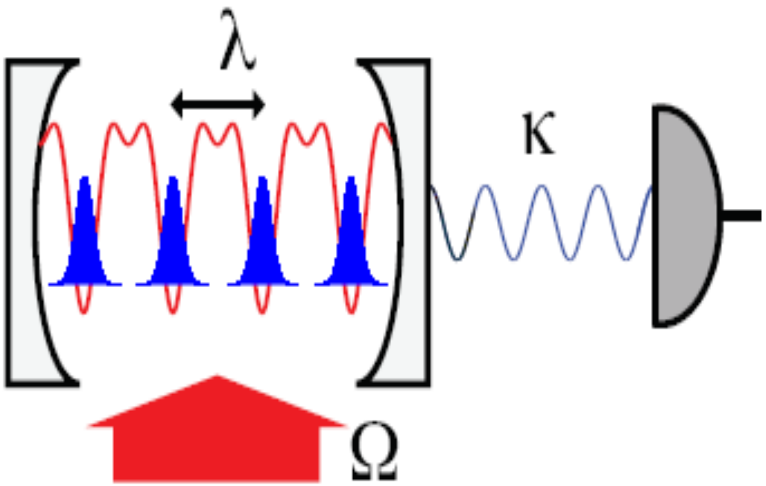
$$\hat{\Phi} = \frac{\sum_l Z_0^{(l)} \hat{n}_l}{K}$$

depends on the density at **all** lattice sites
(Wannier expansion of the order parameter)

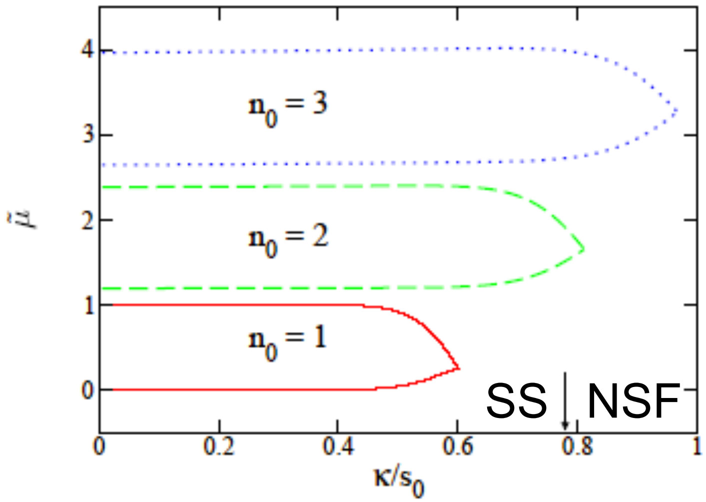
S. Fernandez-Vidal, G. De Chiara, J. Larson, and GM, Phys Rev A 81, 043407 (2010)

Short vs Long range

Insulating phases?



Incompressible states



Pump threshold for self-organization (n=1)

Mott-insulator (checkerboard) patterns manifest due to interplay between onsite and long-range interactions

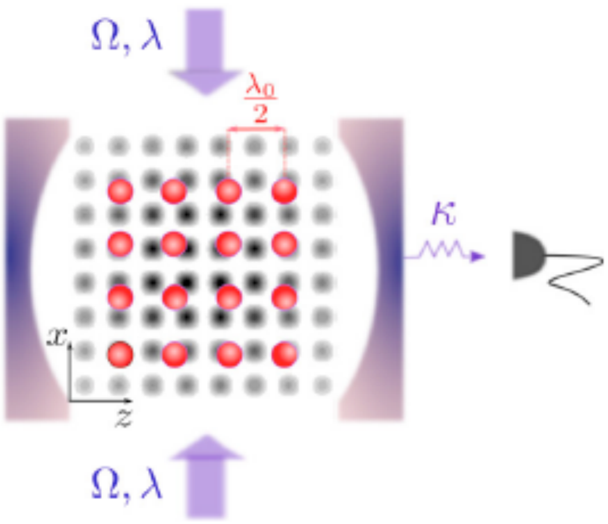
S. Fernandez-Vidal, G. De Chiara, J. Larson, and GM, Phys Rev A 81, 043407 (2010)

Photon-mediated long-range interaction in presence of competing ordering mechanism

Competing orders

Add optical lattice *commensurate* with cavity wave length

$$\lambda = \lambda_0$$



The optical lattice tightly confines the atoms and determines the Wannier functions

Bose-Hubbard model

The optical lattice tightly confines the atoms and determines the Wannier functions
Cavity is a perturbation

$$\hat{\mathcal{H}}_{\mathrm{BH}} = -\sum_{\langle i' j', i j\rangle} t\left(\hat{b}^{\dagger}_{i,j}\hat{b}_{i',j'} + \hat{b}^{\dagger}_{i',j'}\hat{b}_{i,j}\right) + \frac{U}{2}\sum_{i,j}\hat{n}_{i,j}(\hat{n}_{i,j}-1)$$

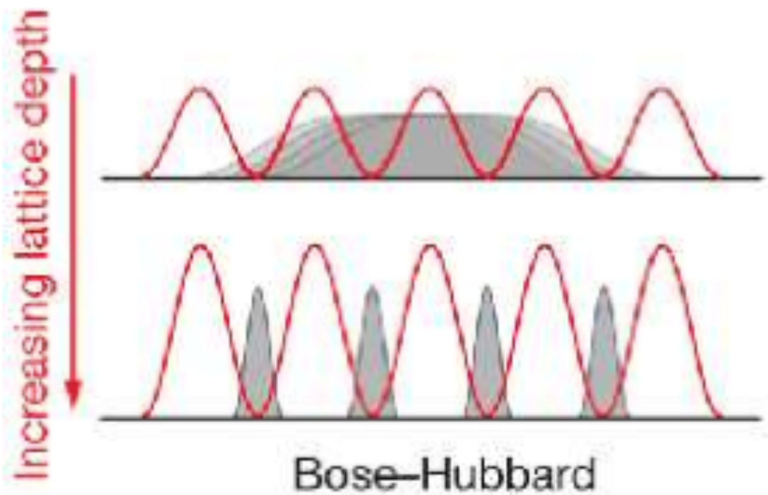
$$-\; V_1 \sum_{i,j} J_0^{(i,j)}\hat{n}_{i,j} + \frac{\hbar s_0^2}{\hat{\delta}_{\mathrm{eff}}^2 + \kappa^2} K\hat{\Phi}^2\left(\delta_c + \hat{\delta}_{\mathrm{eff}}\right)$$

$$\hat{\Phi} = \frac{\sum_l Z_0^{(l)}\hat{n}_l}{K}$$

2D model using local mean-field and cluster analysis

MI-SF phase transition

Superfluid-Mott insulator
ramping the depth of the external optical lattice



deep SF: Uniform distribution

$$\langle \hat{\Phi} \rangle_{\mathrm{SF}} \to 0$$

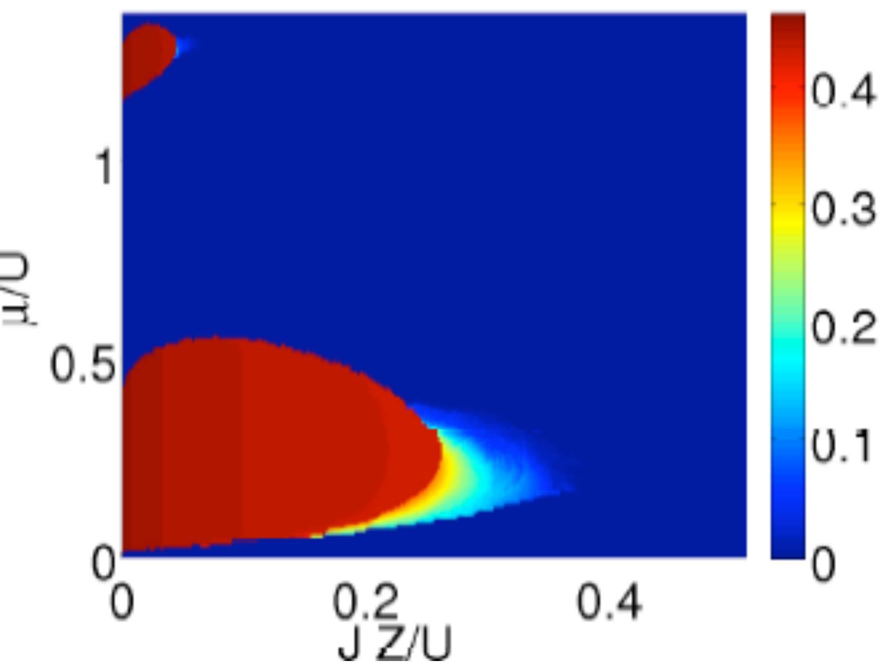
MI: no Bragg scattering

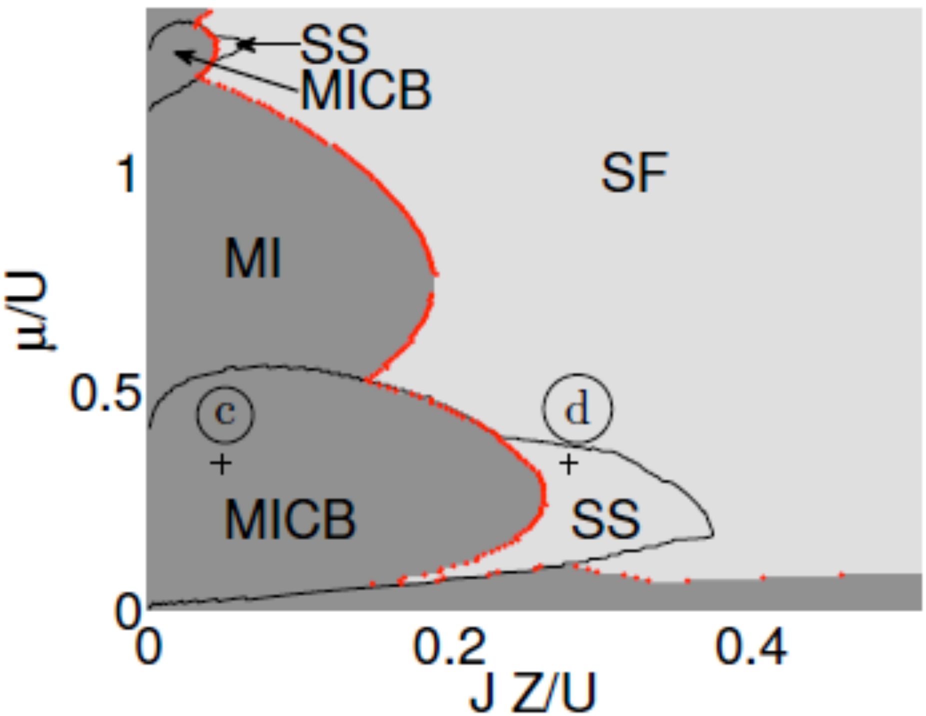
$$\langle \hat{\Phi} \rangle_{\mathrm{MI}} \propto \sum_i Z_0^{(i)} = 0$$

....But (small) quantum fluctuations support the buildup of the cavity field.

Competing orders: phase diagram

Intracavity photon number



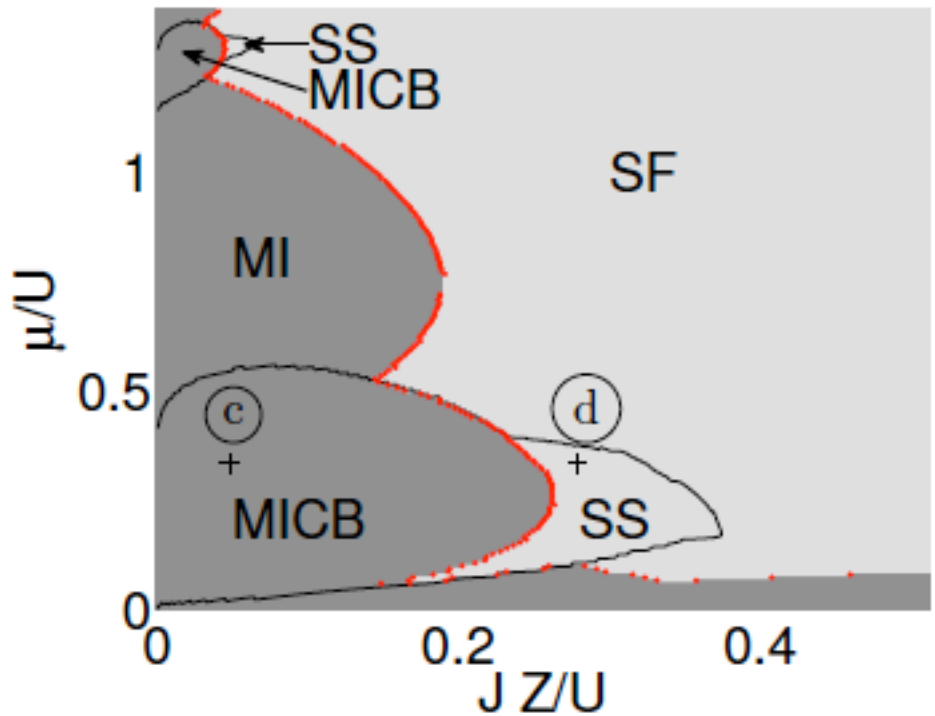


A. Niederle, H. Rieger, GM, preprint (2015)

Competing orders: phase diagram

Identification of 4 phases

	N_{SF}	P	$\hat{\Phi}$
MI	$=0$	$=0$	$=0$
MICB	$=0$	$=0$	>0
SF	>0	$=1$	$=0$
SS	>0	$=1$	>0

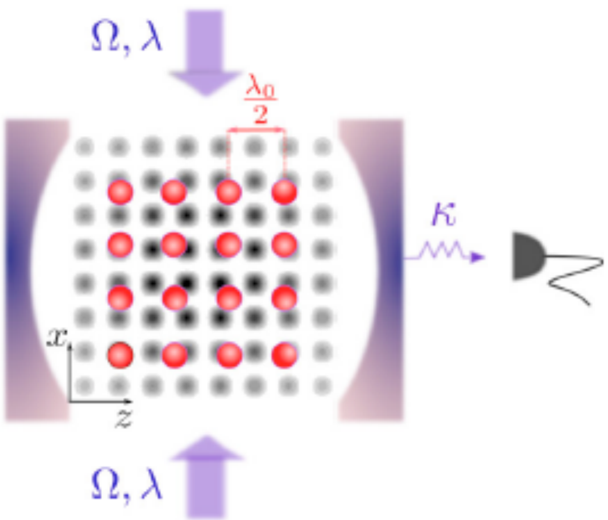


A. Niederle, H. Rieger, GM, preprint (2015)

Frustration

Optical lattice is incommensurate
with cavity wave length

$$\lambda = \lambda_0 + \epsilon_\lambda$$

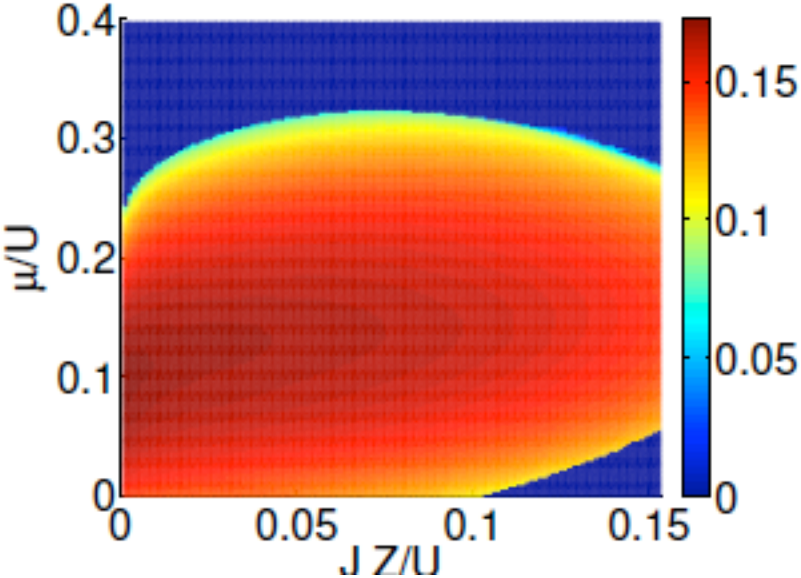


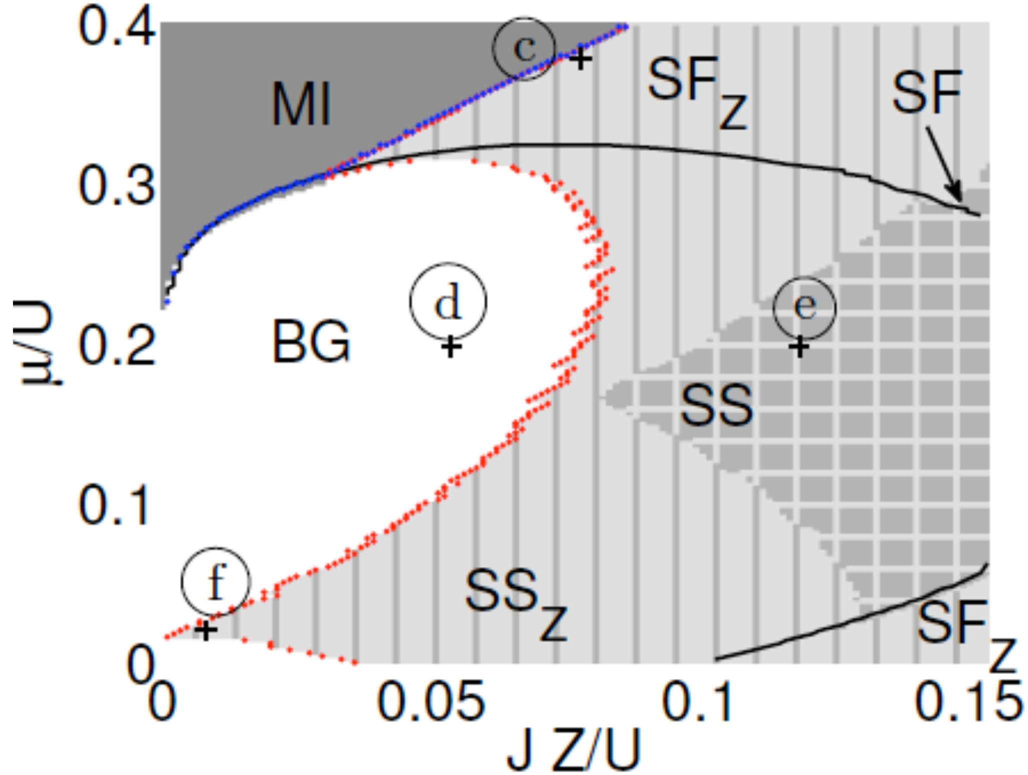
The optical lattice tightly confines the atoms and determines the Wannier functions

2D model using local mean-field and cluster analysis

Frustration and Bragg order

Number of photons



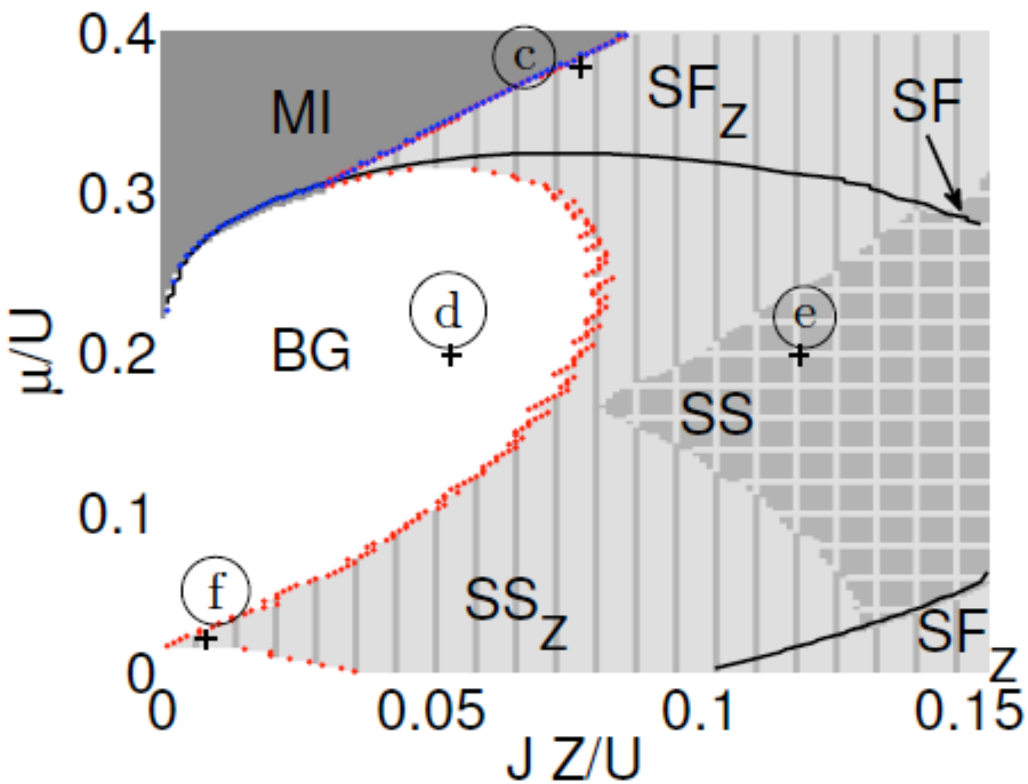


A. Niederle, H. Rieger, GM, preprint (2015)

Frustration and Bragg order: Phase diagram I

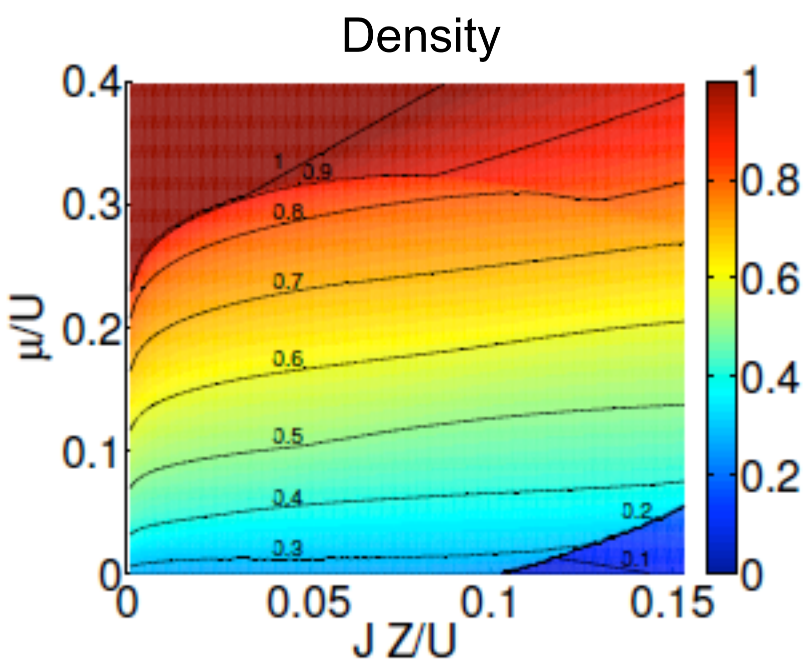
Identification of 8 phases

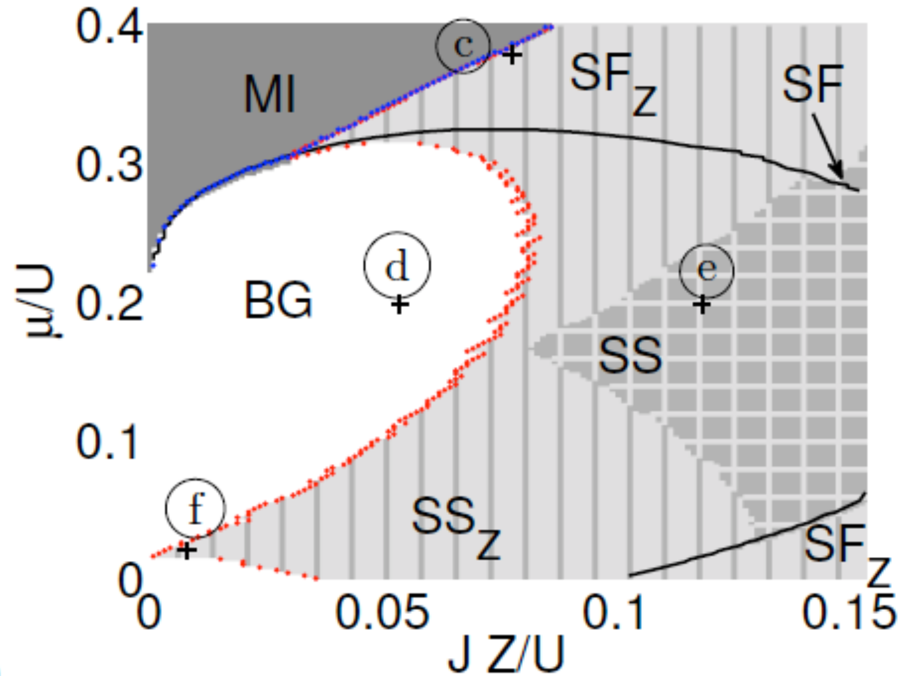
	N_{SF}	P_X	P_Z	$\hat{\Phi}$
MI	$=0$	$=0$	$=0$	$=0$
BG	>0	$=0$	$=0$	>0
SF	>0	$=1$	$=1$	$=0$
SF_X	>0	$=1$	$=0$	$=0$
SF_Z	>0	$=0$	$=1$	$=0$
SS	>0	$=1$	$=1$	>0
SS_X	>0	$=1$	$=0$	>0
SS_Z	>0	$=0$	$=1$	>0



A. Niederle, H. Rieger, GM, preprint (2015)

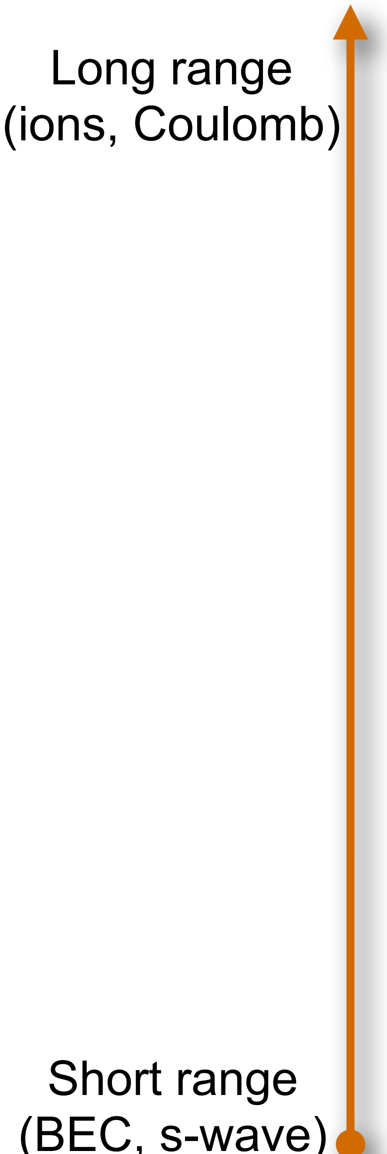
Frustration and Bragg order: Phase diagram II





Density and onsite interactions determine the resulting pattern

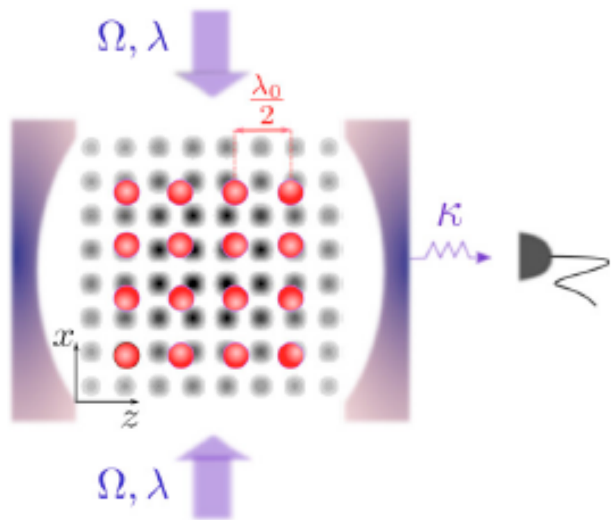
A. Niederle, H. Rieger, GM, preprint (2015)


Long range
(ions, Coulomb)
Short range
(BEC, s-wave)

Long range
(ions, Coulomb)

Short range
(BEC, s-wave)

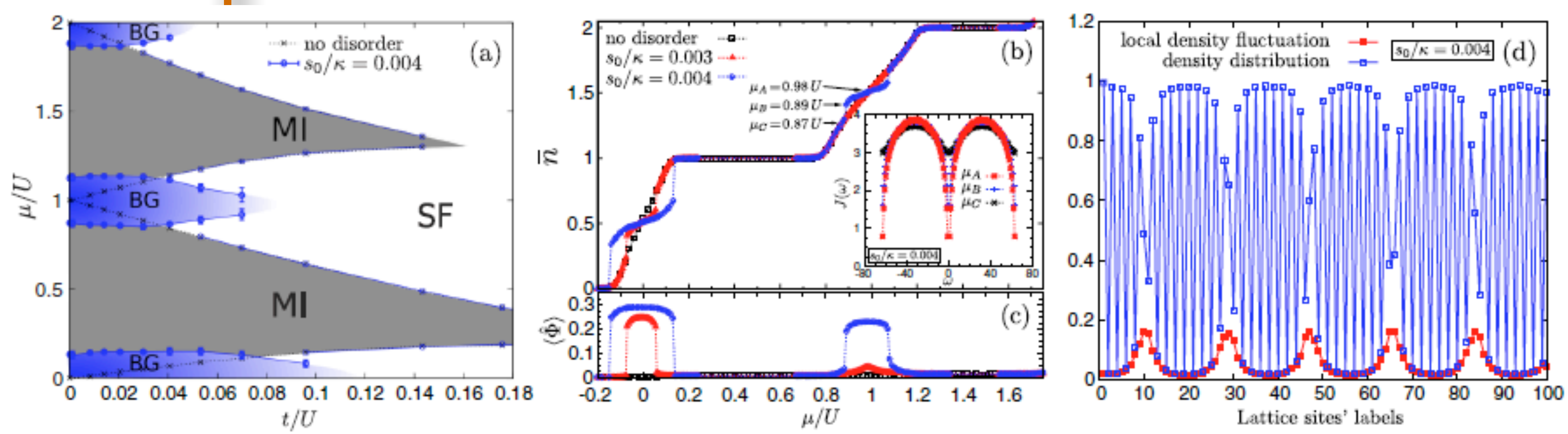
Optical lattice incommensurate
with cavity wave length



regime where retardation can be neglected
(Hamiltonian dynamics)

Long range
(ions, Coulomb)

Exotic phases due to cavity back-action



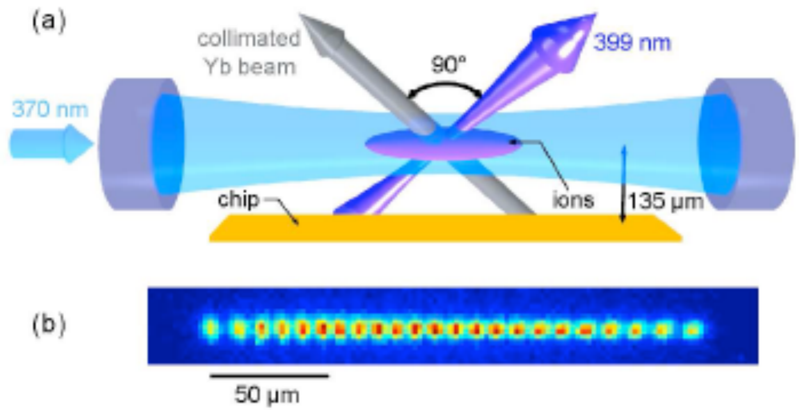
Short range
(BEC, s-wave)

H. Habibian, A. Winter, S. Paganelli, H. Rieger, GM, Phys Rev. Lett 110, 075304 (2013)

Ion crystal in a cavity

Long range
(ions, Coulomb)

Short range
(BEC, s-wave)

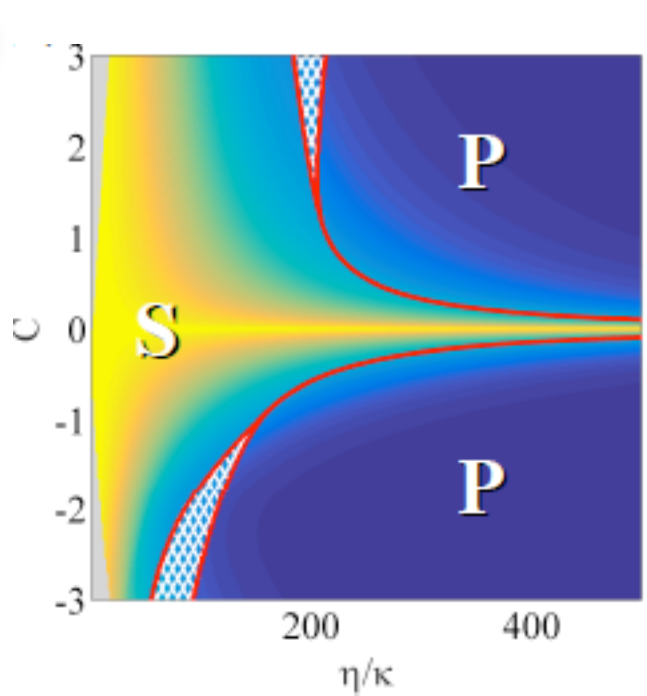


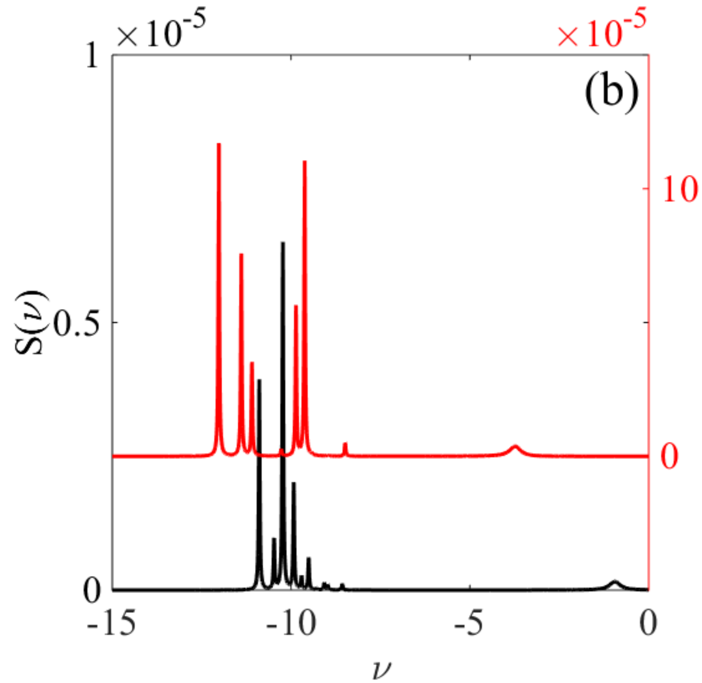
Typical length of crystallization is incommensurate with cavity wave length

(figure from Cetina et al, NJP 2013)

Exotic model of friction

Long range
(ions, Coulomb)





Short range
(BEC, s-wave)

T. Fogarty, C. Cormick, H. Landa, V. M. Stojanovic, E. A. Demler, GM, arXiv:1504.00265

Outlooks

- Phase diagram showing the interplay between short and long range mechanism
- Quenches: Kibble-Zurek paradigma for long-range interacting potentials?
- Many-Body physics in CQED: exciting platform for studying the physics of long-range interactions

Thanks to....

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